Online Appendix

Monetary Policy and Inequality under Labor Market Frictions and Capital-Skill Complementarity

Juan Dolado, Gergő Motyovszki, Evi Pappa [1]

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Abstract

In this Appendix we present robustness checks for the empirical specification we use in the main text as well as sensitivity analysis for the results presented in the theoretical model. We further present the responses of the model economy to other shocks and a complete description of the theoretical model, its steady state and derivations of the wage dynamic decomposition.

A Alternative SVARs

A.1 Extended sample

The Romer and Romer narrative shocks series, on which we base identification of monetary policy shocks using the IV-SVAR approach, ends in 2007. However, since our labor market data extends to 2016, we report here results to improve inference using the longer narrative series constructed by Miranda-Agrippino (2016) up to 2012, despite the concerns in Ramey (2016) about the existence of traditional monetary policy shocks during the ZLB period. Figures 1 (for the aggregate economy) and 2 (for the sectors) show that results are robust for that extended sample.

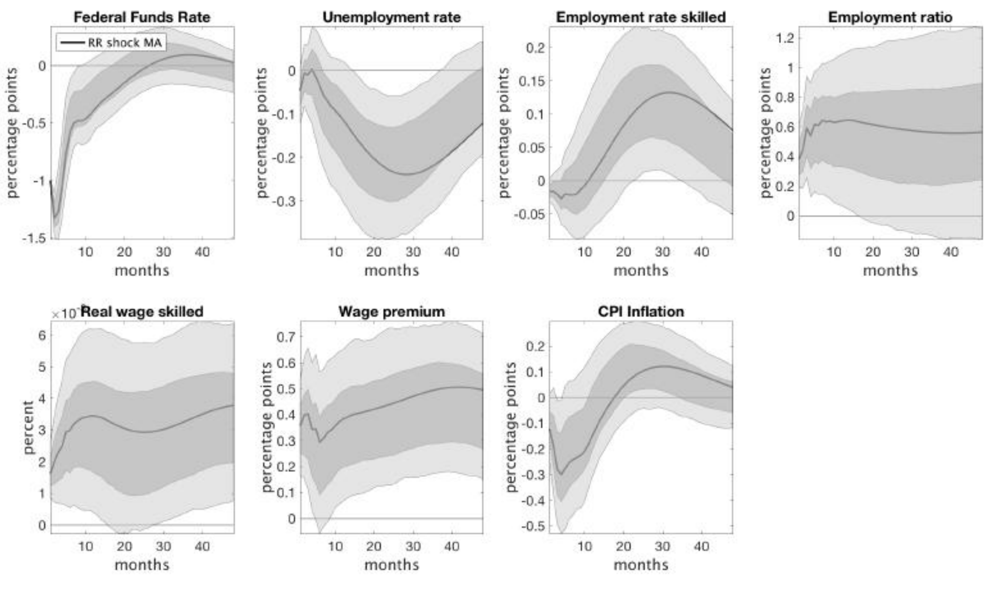


Figure 1: IRFs to an (annualised) 100 bp. unexpected reduction in the FF interest rate - extended sample 1980-2012. The labor market series are averaged using a 5-month backward moving average.

[1]European University Institute & Universidad Carlos III de Madrid. Compiled on Wednesday 5$^{\text{th}}$ February, 2020, at 23:30

A.2 Cholesky in whole sample

Despite believing that the shorter sample 1980-2007 used in the Proxy-SVAR, is the correct one for the identification of meaningful monetary policy shocks, we present here IRFs of a SVAR for the extended sample up to 2016, i.e. the period for which the labor market series are available. Instead of using an IV-SVAR, this time monetary policy shocks are recovered from a lower triangular Cholesky decomposition. The identifying assumptions are that the FFR is allowed to respond on impact to shocks in all remaining variables, while real variables and prices do not react to FFR shocks within a month. The IRFs for this specification are presented in Figure 3. The main finding is that the monetary policy shocks we recover have qualitatively similar effects on the labor market variables as the ones recovered from our IV-SVAR exercise for the shorter sample. The only exception is that the IRF of inflation is negative and statistically significant after the monetary expansion (the price puzzle is present).

A.3 Local projections

To examine the robustness of our baseline results, we have also analyzed how a monetary policy shock affects the skill premium and relative employment when using direct local projections (LP). Plagborg-Moller and Wolf (n.d.) shows that LP and VARs estimate the same IRFs for unrestricted lag structures. This would be true also in our model at monthly frequency. Yet, since Coibion et al. (2012) use data at the quarterly frequency showing that monetary contractions increase inequality, we have replicated their results in exercises we do not present here to save space. In addition, we analyze how monetary shocks affect the skill premium and relative employment transforming our data to quarterly frequency. In particular, in performing this exercise, we follow as closely as possible the methodology used by Coibon et al. (2017). In particular, we have converted the monthly series into quarterly by taking the observation of the middle month of the quarter as the quarterly observation for real wages and employment, and focus on a monetary contraction, in line with these authors. Like them, we use LP with 2 lags for the endogenous variables and 20 lags for the shocks series and perform estimation between 1980Q1 and 2008Q4. Figure 4 presents mean responses and 90 percent confidence bands for relative employment and the skill premium, as well as for the (logged) levels of employment and real wage of skilled workers in response to a monetary *contraction*.

The main finding is that the IRFs plotted in Figure 4 are qualitatively similar to the ones obtained in our Proxy-SVAR exercise.

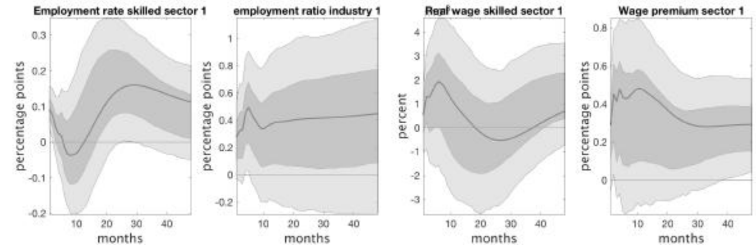
A.4 Composition effects

Measured wage fluctuations over the business cycle can be heavily affected by composition effects. To make sure our results are not driven by these effects, we have constructed alternative series for the repeated (monthly) cross sections of hourly wages as the residuals in a time series of cross-sections regression of wages on a number of controls. In particular, we have regressed the (logged) hourly wage – separately for each of the two worker categories – on socio-demographic controls different from education (i.e. age group, gender, race, and marital status) and dummies for each state. We then averaged the residuals for each month once they have been seasonally adjusted and deflated by the CPI. We have then defined the skill premium as the difference in the residuals for high and less-skilled workers.

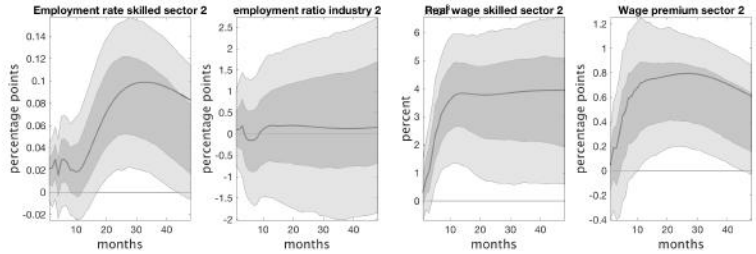
Figure 5 displays below the IRFs with the newly constructed wage series. As can be observed, our baseline results remain robust after controlling for composition effects.

A.5 Raw series of labor market data

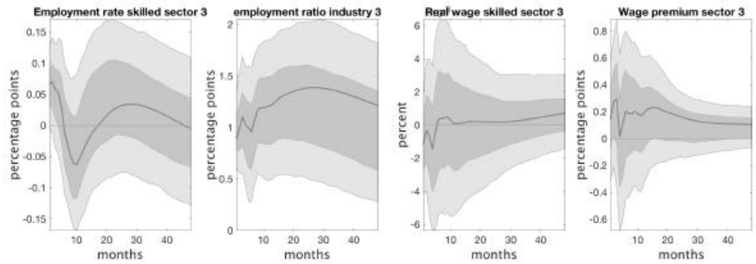
The series of wages and employment we recover from the repeated cross sections of the CPS turn out to be highly volatile. Economists often use a "moving average" to smooth the data. However, there is a downside to using a centered moving average as a smoothing device since the calculation relies on historical data and some of the variable's timeliness is lost. This is especially relevant when one wants to introduce the data series in a VAR. By definition, in an auto-regressive model each variable (time series) is modeled as a function of the past values of the series. As a result, if one uses a moving average to smooth the data it needs to be a backward moving average. In the main text we present results when we average the labor market series using a 5-month backward moving average which also corresponds to the optimal lag length of the VAR according to the conventional information criteria. In Figure 6 we present results from our baseline specification when we use the raw labor market series in the VAR. Although responses differ quantitatively, our main result (i.e. a monetary expansion induces a significant increase in the skill premium and relative employment) remains robust.



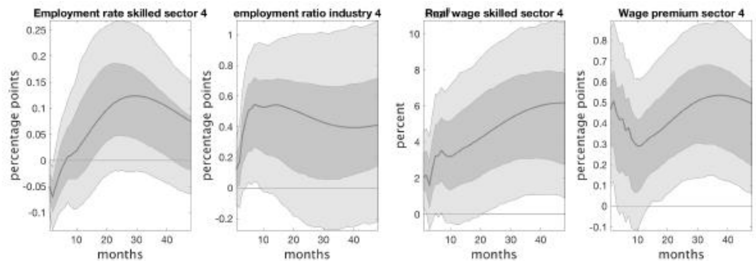
(a) Manufacturing



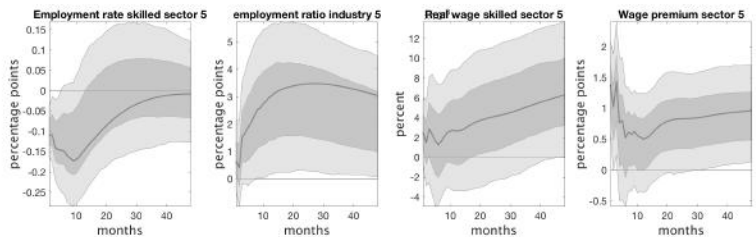
(b) Education and Health Services



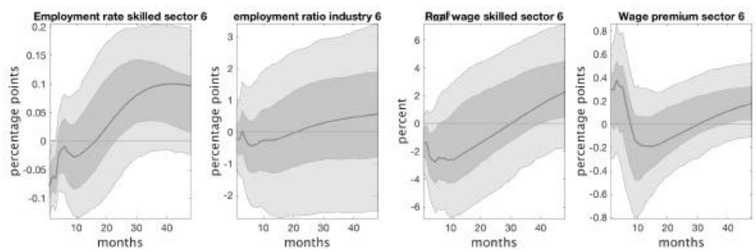
(c) Agriculture, Mining and Transportation



(d) Wholesale and Retail Trade



(e) Professional Services



(f) Financial and Informational Services

Figure 2: IRFs of labor market variables in different sectors to an (annualised) 100 bp. unexpected reduction in the FF interest rate - extended sample 1980 -2012. The labor market series are averaged using a 5-month backward moving average.

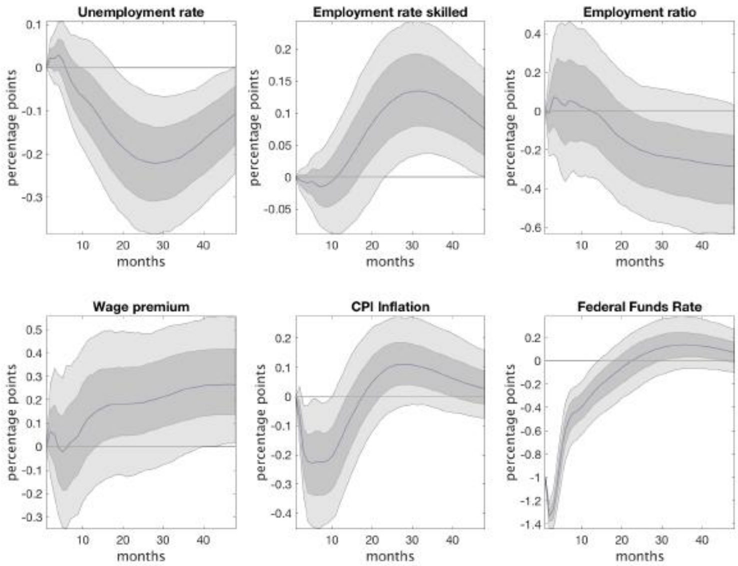
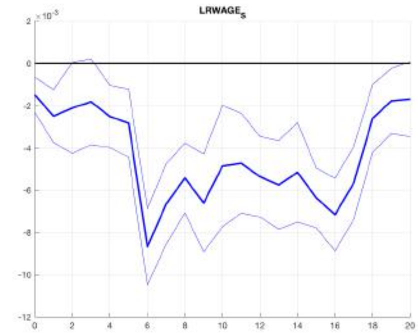
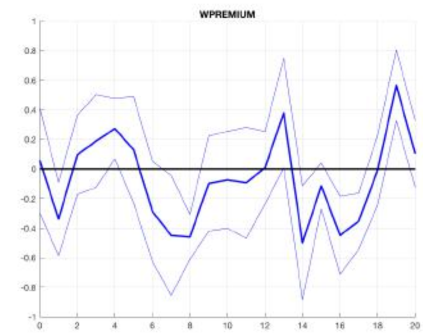


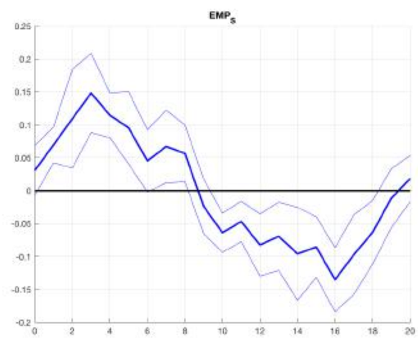
Figure 3: IRFs to a one percentage point unexpected reduction in the FF interest rate - Cholesky decomposition 1980-2016. The labor market series are averaged using a 5-month backward moving average.



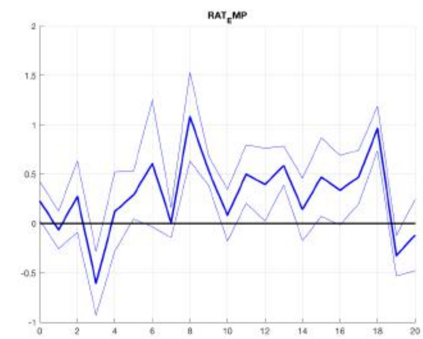
(a) Wages of skilled



(b) Wage Premium



(c) Skilled employment rate



(d) Relative employment

Figure 4: Responses using local projections

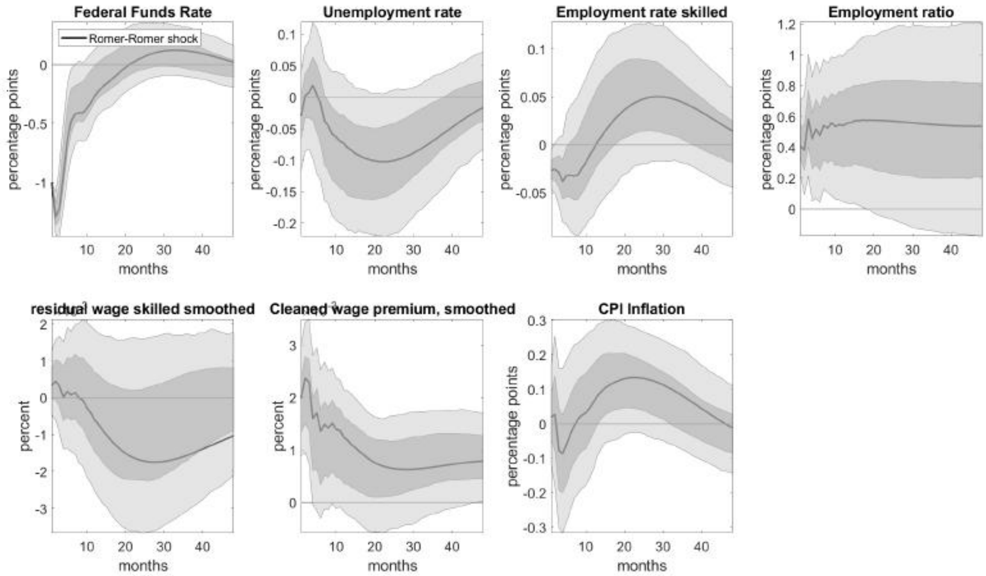


Figure 5: IRFs with series averaged over the 5 previous months

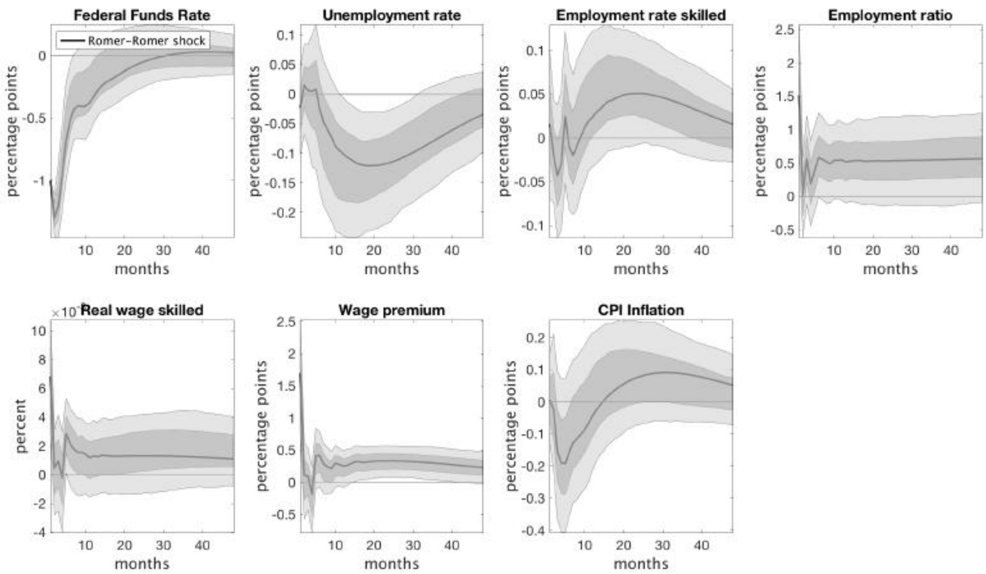


Figure 6: Responses of labor market series when we control for wage composition effects

B Further IRFs – Monetary policy shock

B.1 Sensitivity analysis for key parameters

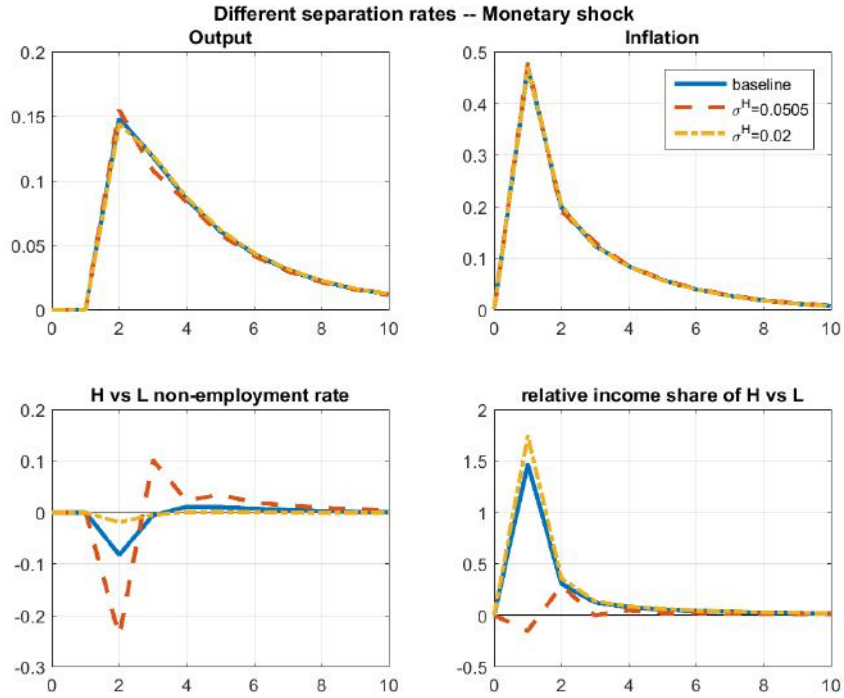


Figure 7: IRFs after an (annualized) 100 bps cut in the policy interest rate. Baseline is $\sigma^H = 0.0299, \sigma^L = 0.0505$.

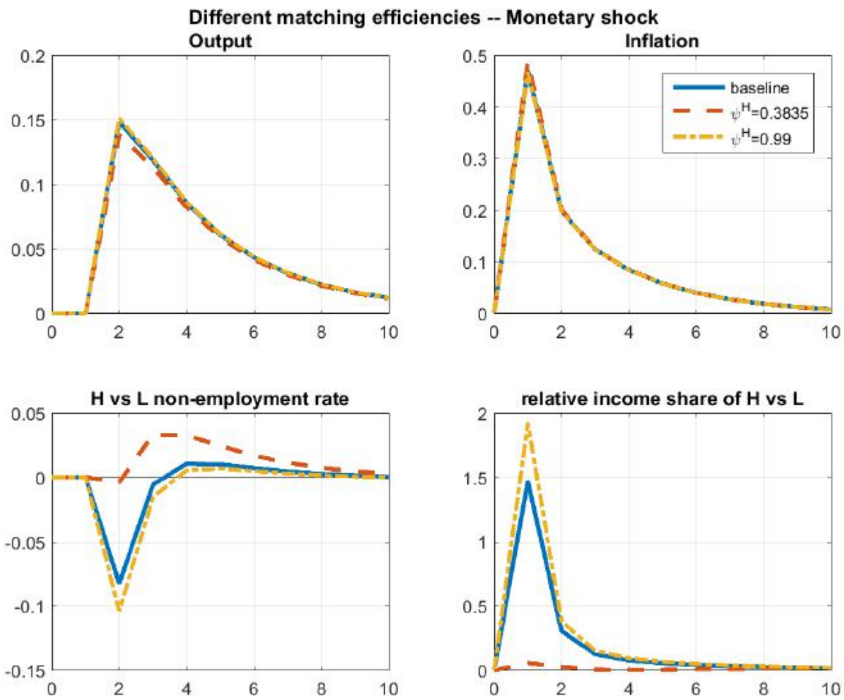


Figure 8: IRFs after an (annualized) 100 bps cut in the policy interest rate. Baseline is $\psi^H = 0.8630, \psi^L = 0.3835$.

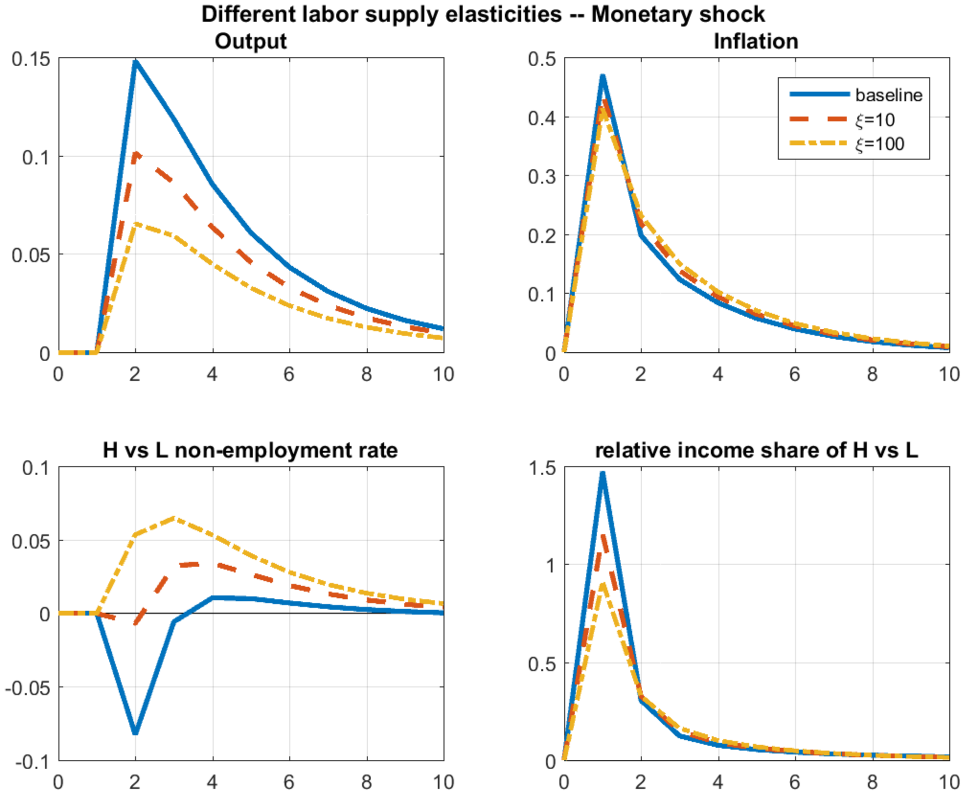


Figure 9: IRFs after an (annualized) 100 bps cut in the policy interest rate. Baseline is $\xi = 4$.

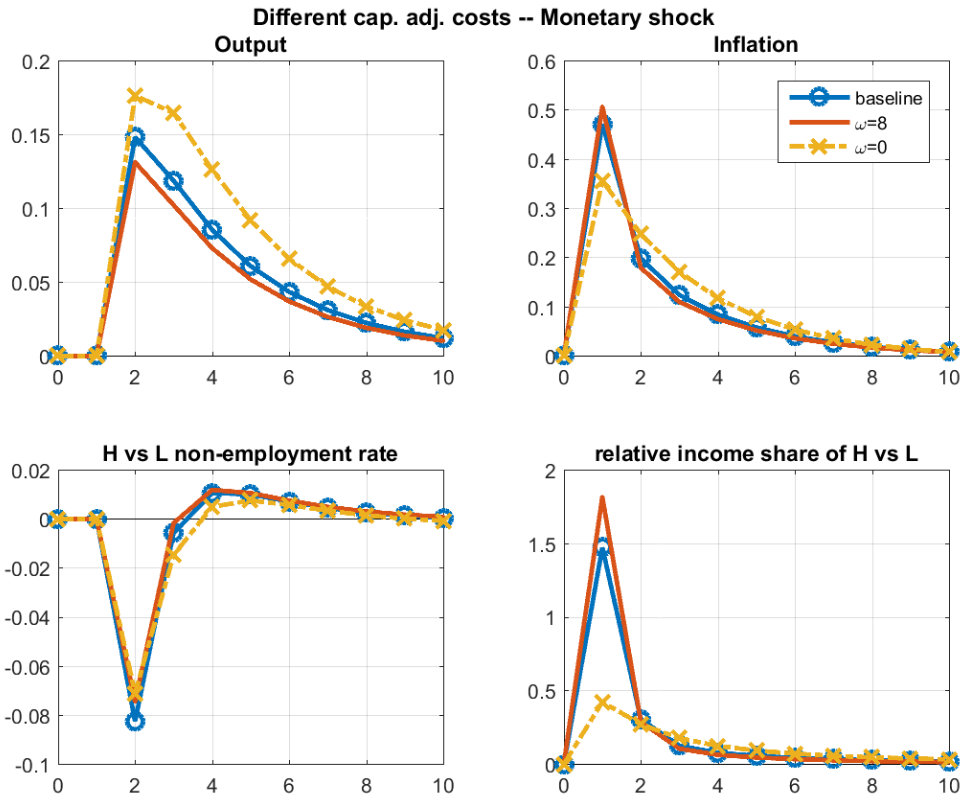


Figure 10: IRFs after an (annualized) 100 bps cut in the policy interest rate. Baseline is $\omega = 4$.

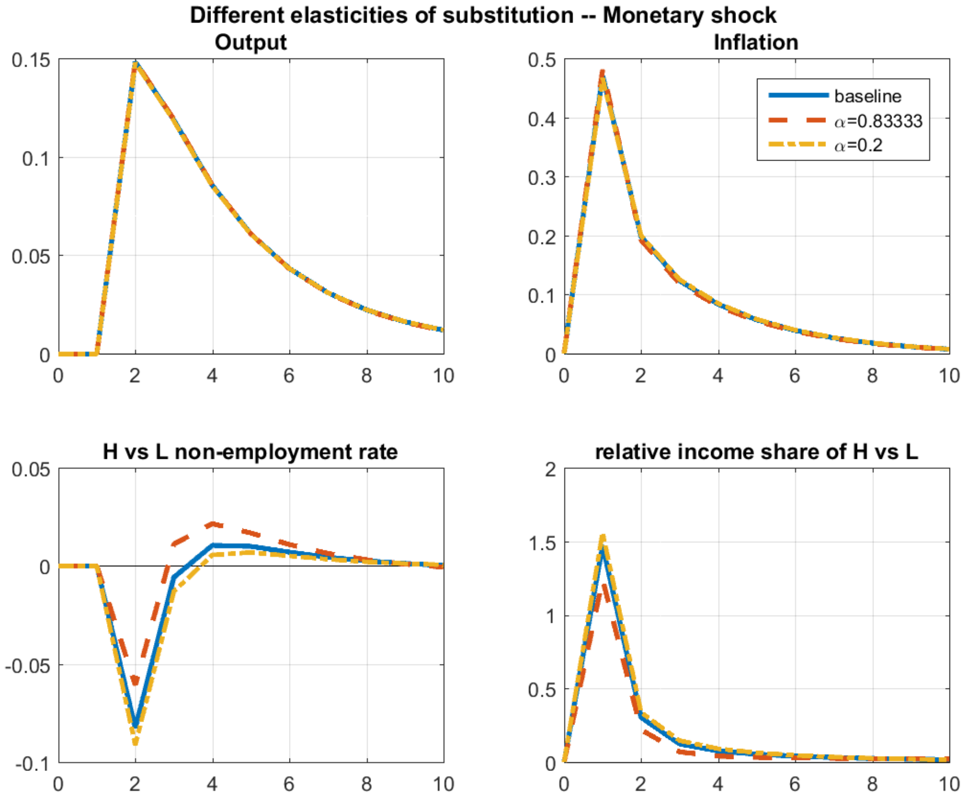


Figure 11: IRFs after an (annualized) 100 bps cut in the policy interest rate. Baseline is $\alpha = 0.4$.

B.2 Rigid wages

In the baseline model we assume wages are flexible, yet, real wage rigidity might affect the reaction of the wage premium to a monetary policy shock. In this section we investigate how the responses of the relative income share change when we assume some wage rigidity. In particular, current wages w_t are assumed to follow a partial adjustment model: $w_t^k = \rho_w^k w_{t-1}^k + (1-\rho_w^k) w_t^{k*}$, which is meant to be a shortcut for wage rigidities, controlled by parameter ρ_w^k, $k = H, L$. In Figure 12 we display responses when we assume wage rigidities for both type of workers $\rho_w^H = \rho_w^L = 0.8$ and when we assume wages are more rigid for less-skilled workers, i.e., $\rho_w^H = 0.6$ and $\rho_w^L = 0.8$.

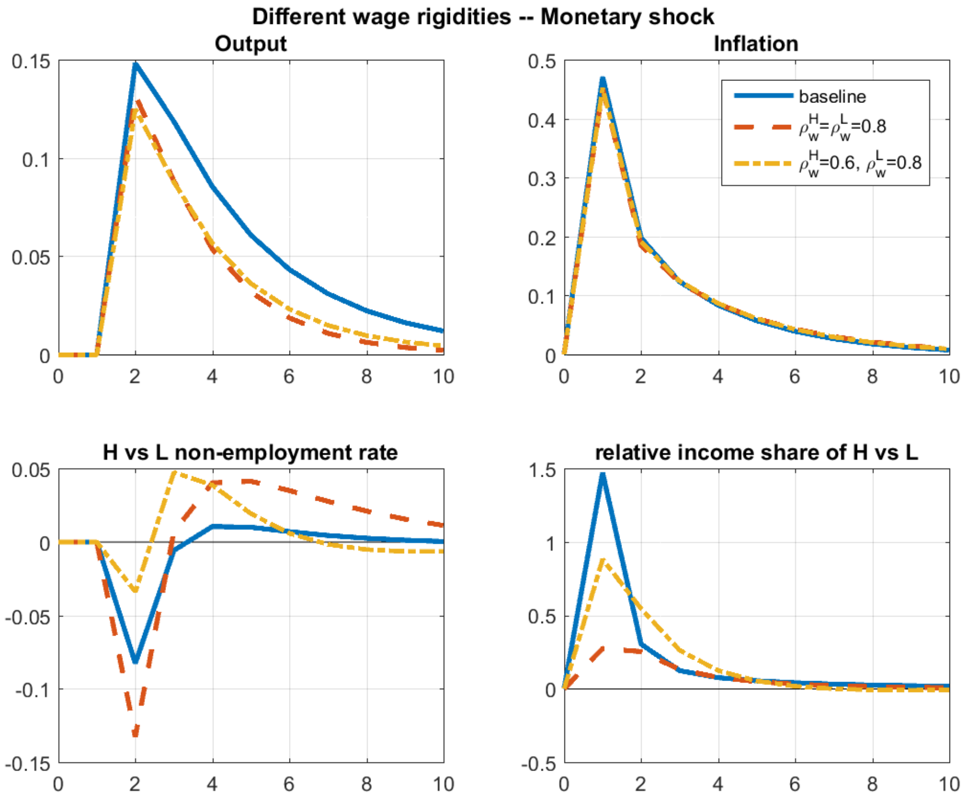


Figure 12: IRFs after an (annualized) 100 bps cut in the policy interest rate. Baseline is $\rho_w^H = \rho_w^L = 0$.

B.3 Variable capital utilization

In this section we present IRFs when we allow for variable capital utilization in the baseline model. In this case, both effective capital and investment expand on impact after the shock, in contrast to what happens without such assumption. Yet, the responses of real marginal costs and the respective steady state coefficients that determine the skill premium responses are very similar to those presented in the main text.

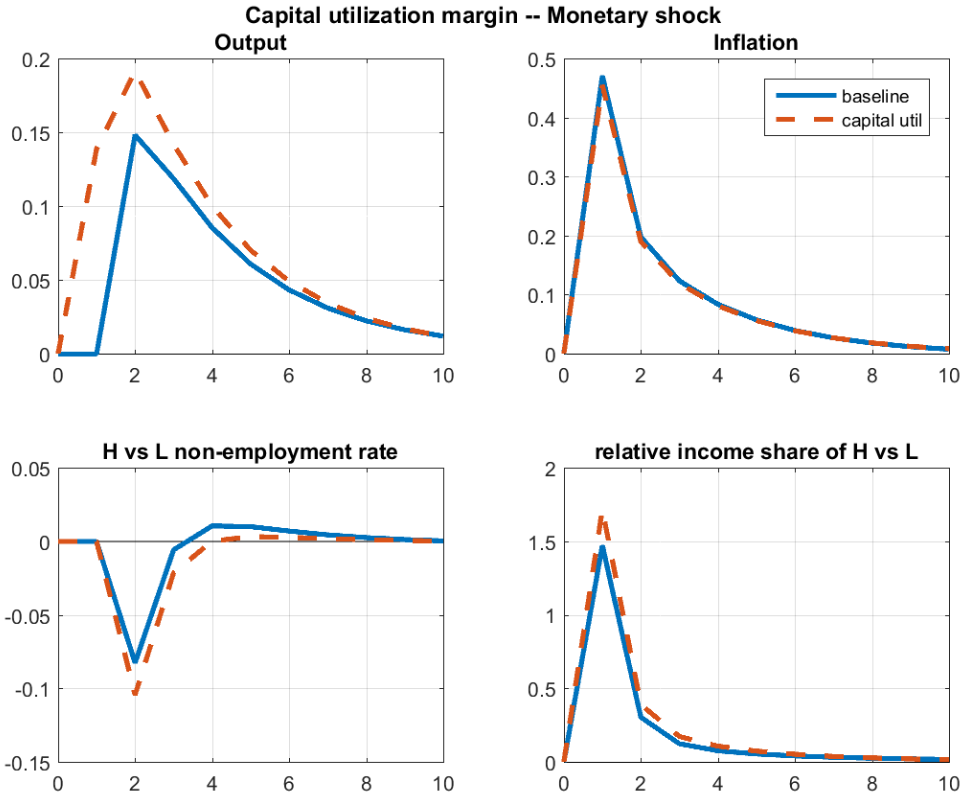


Figure 13: IRFs after an (annualized) 100 bps cut in the policy interest rate. Baseline is without capital utilization margin.

B.4 Different monetary policy rules

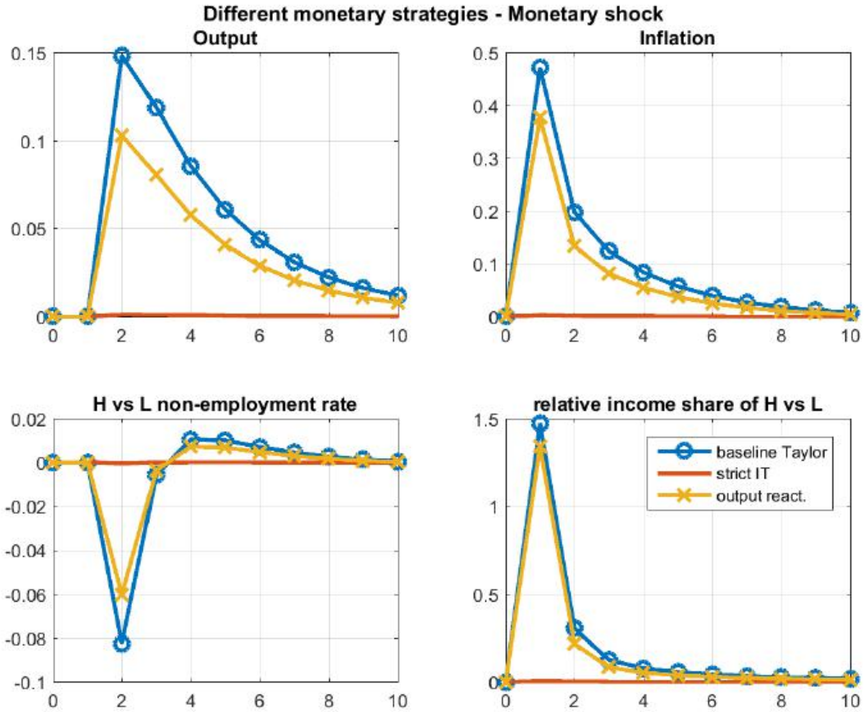


Figure 14: IRFs after an (annualized) 100 bps cut in the policy interest rate. Baseline is $\zeta^y = 0, \zeta^\pi = 1.5$.

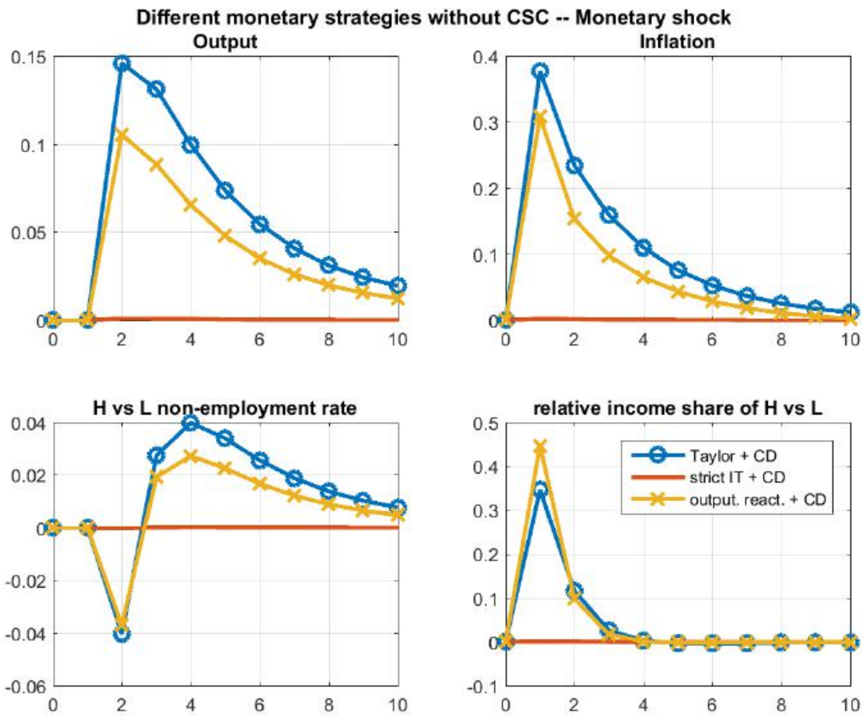


Figure 15: IRFs after an (annualized) 100 bps cut in the policy interest rate. Baseline is $\zeta^y = 0, \zeta^\pi = 1.5$.

C Further IRFs – Other shocks

C.1 Expansionary government spending shock

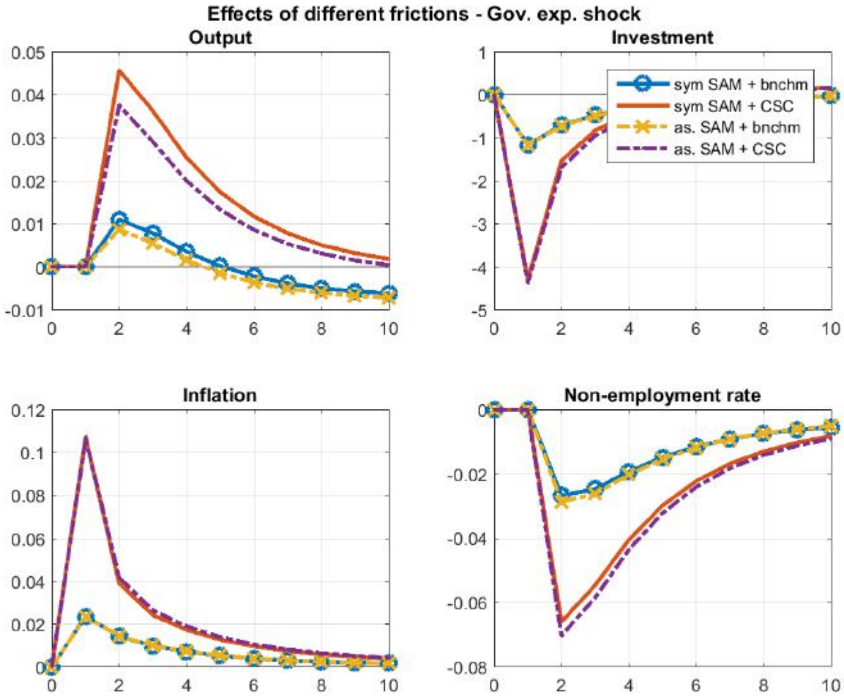


Figure 16: IRFs after a 1% increase in G_t

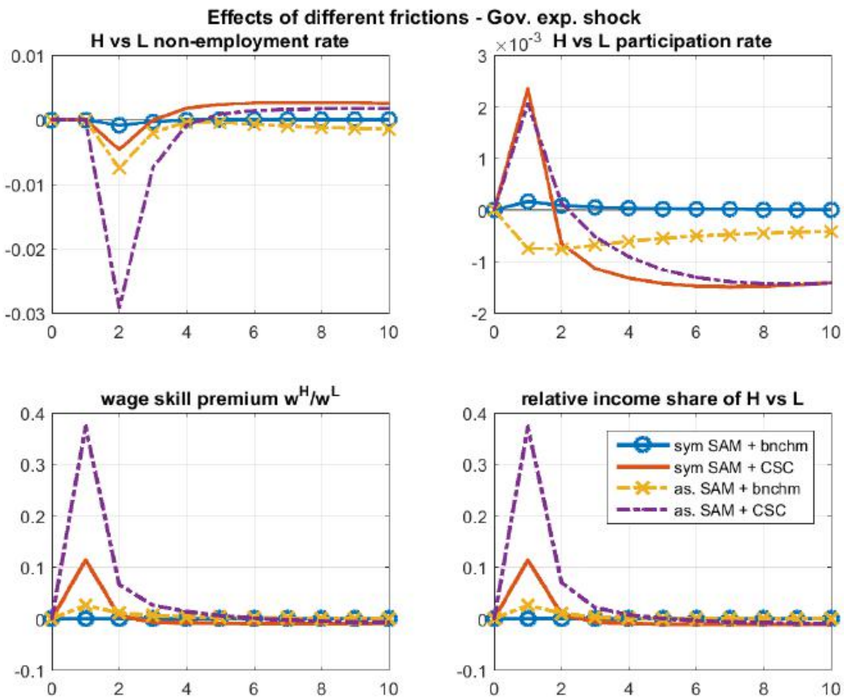


Figure 17: IRFs after a 1% increase in G_t

C.2 Investment price shock (negative)

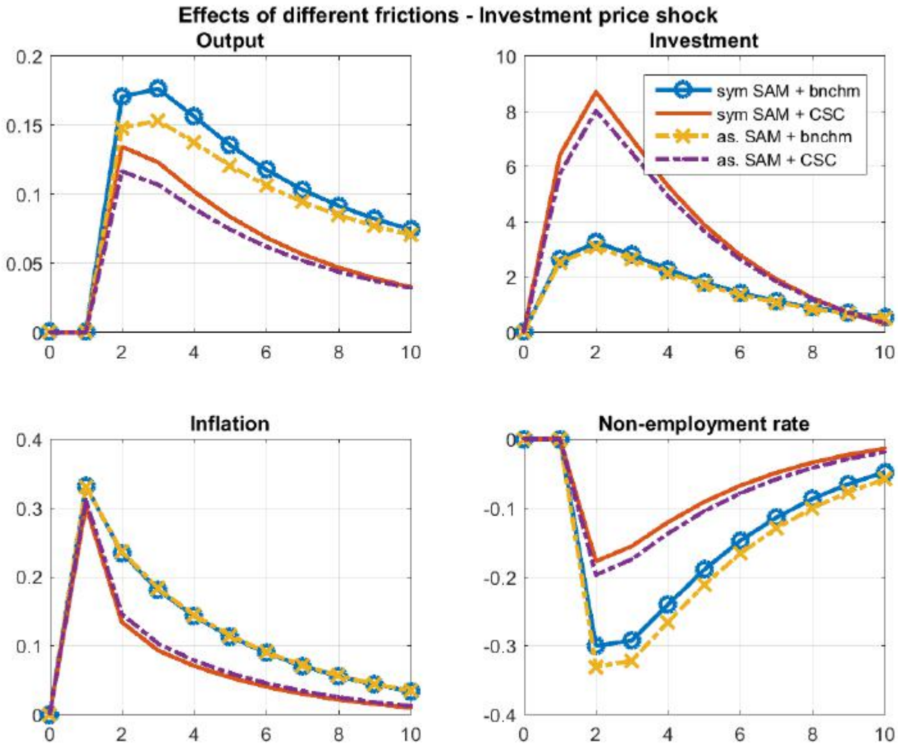


Figure 18: IRFs after a 1% decrease in the price of investment goods

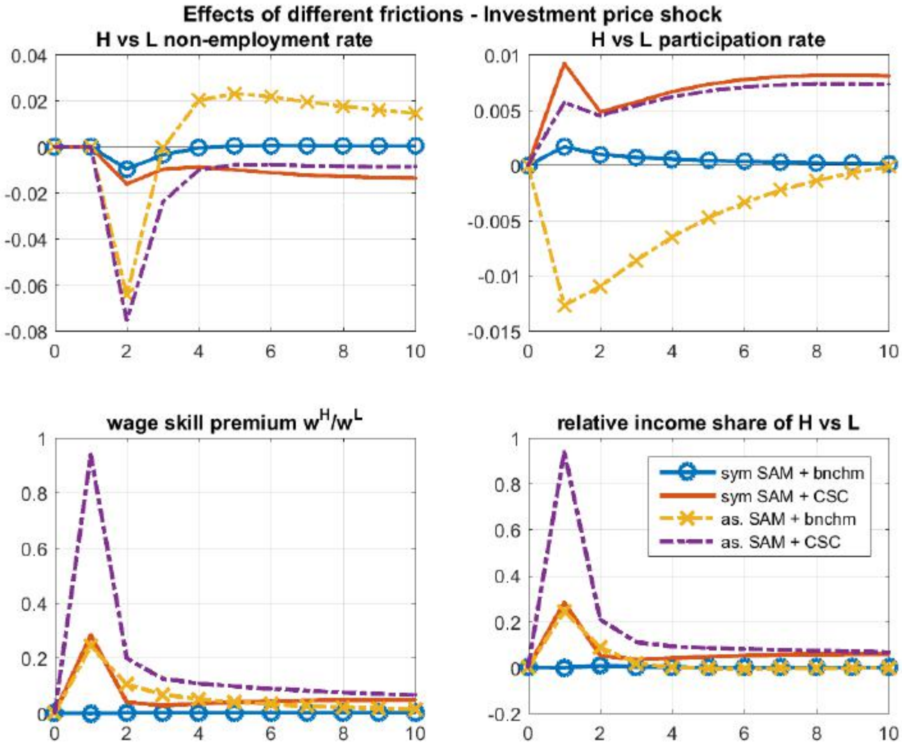


Figure 19: IRFs after a 1% decrease in the price of investment goods

C.3 Favorable (negative) cost-push shock

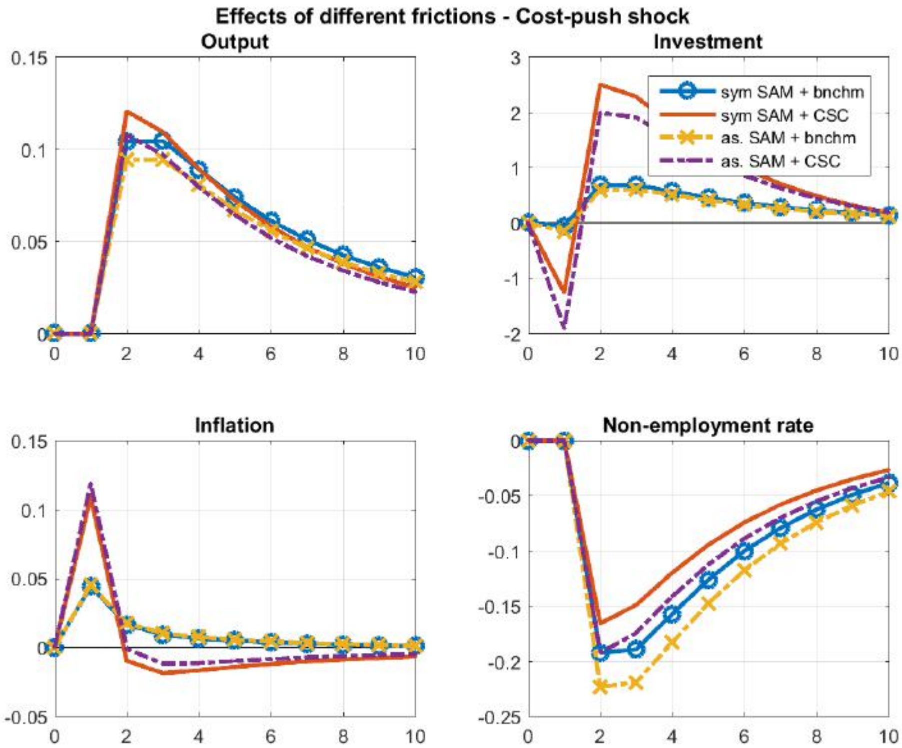


Figure 20: IRFs after a 1% decrease in Ξ_t

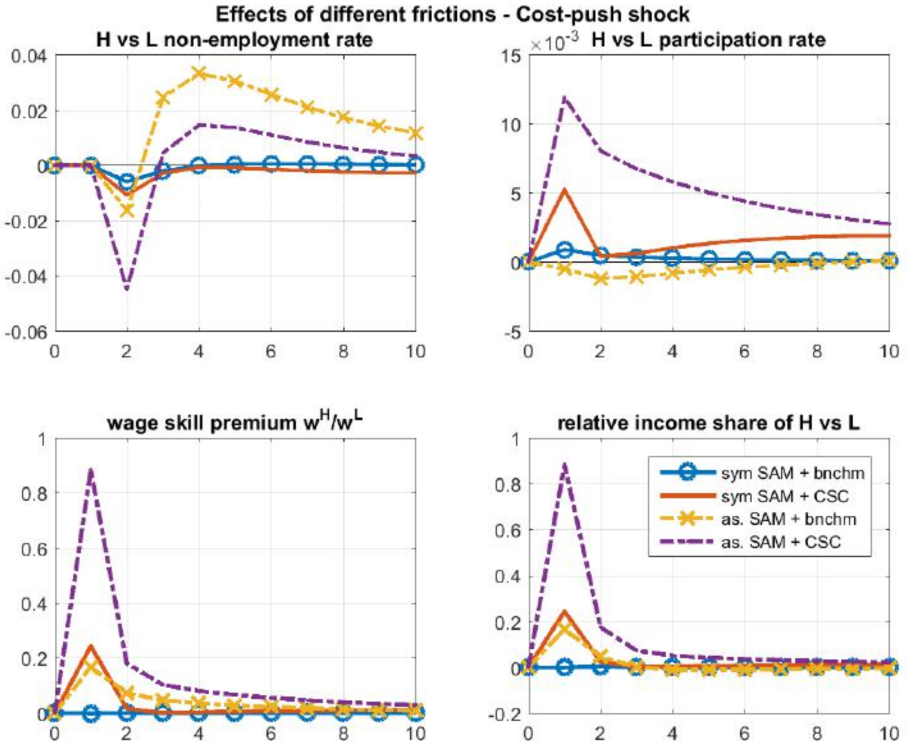


Figure 21: IRFs after a 1% decrease in Ξ_t

C.4 Favorable cost-push shocks and monetary policy rules

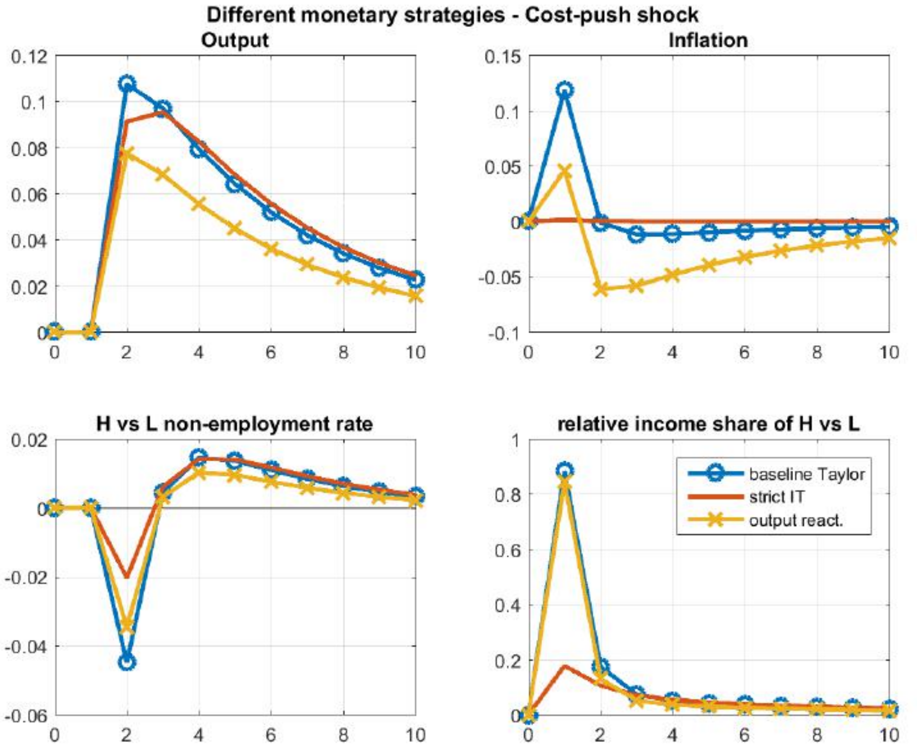


Figure 22: IRFs to a favourable cost push shock under different monetary policy rules – *with* CSC

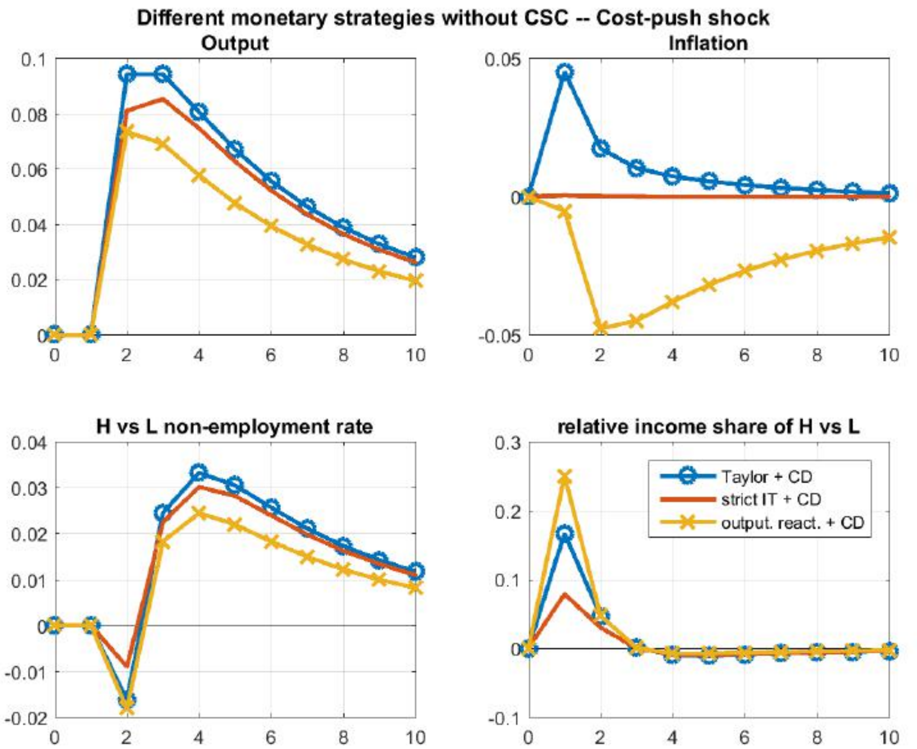


Figure 23: IRFs to a favourable cost push shock under different monetary policy rules – *without* CSC

C.5 Negative (expansionary) discount factor shock

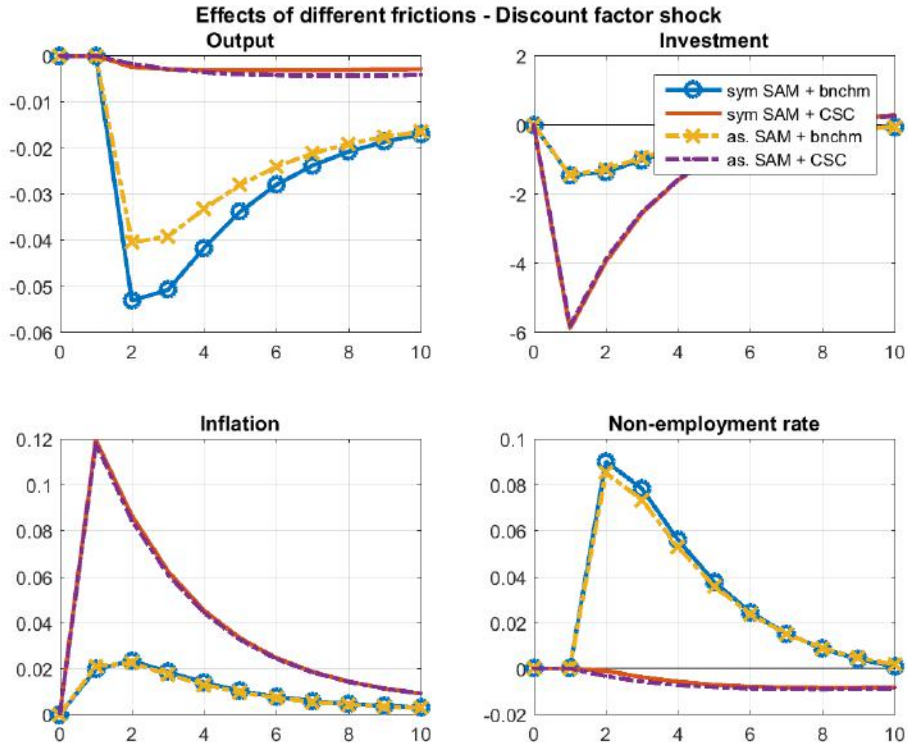


Figure 24: IRFs after a 1% decrease in Ω_t

C.6 Positive TFP shock

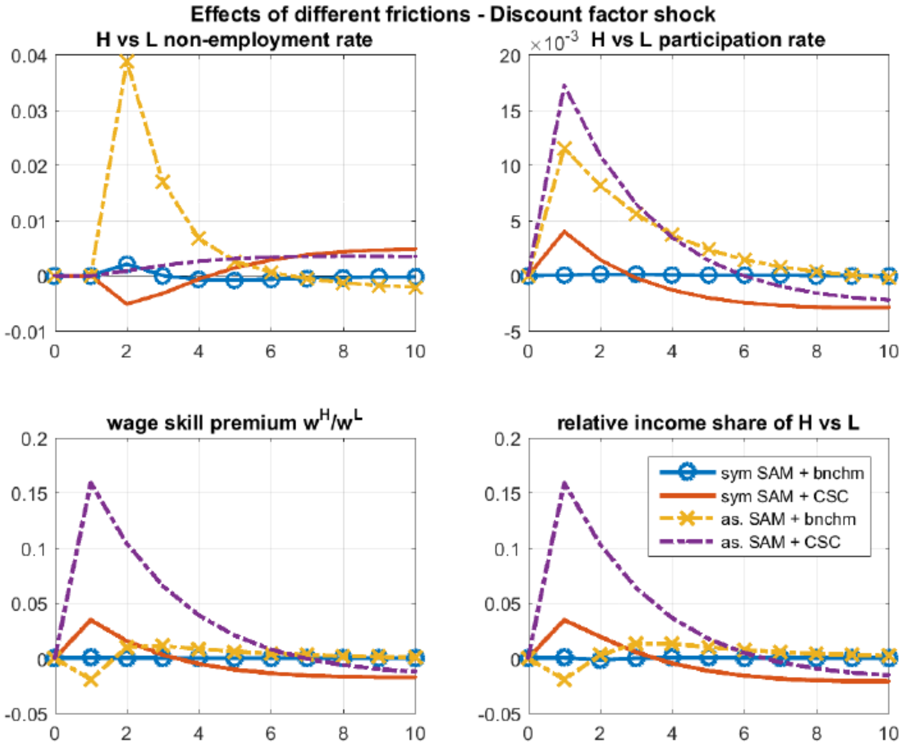


Figure 25: IRFs after a 1% decrease in Ω_t

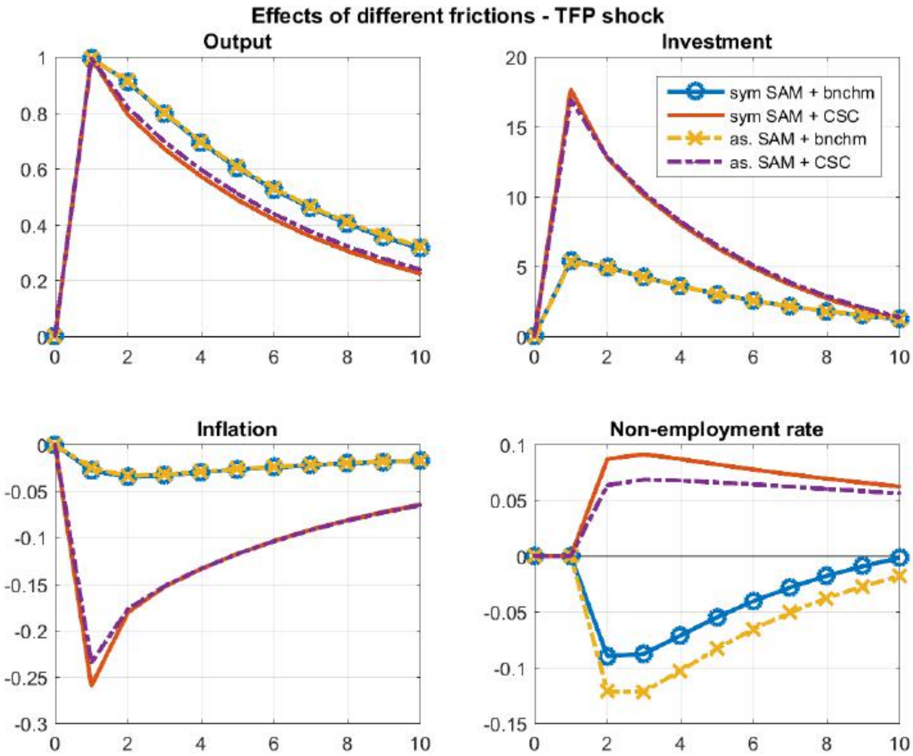


Figure 26: IRFs after a 1% increase in A_t

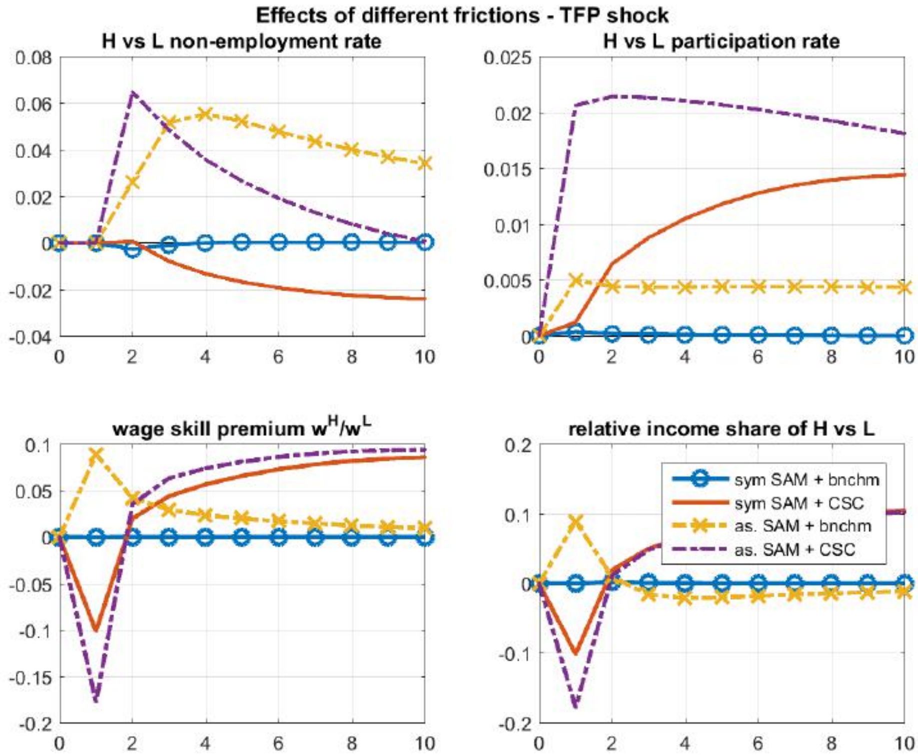


Figure 27: IRFs after a 1% increase in A_t

D Equilibrium conditions

D.1 Dynamic (non-linear)

In the sequel, we present a more general version of our model from the main text, featuring incomplete markets between different household types (but maintaining full insurance within types). We also add variable capital utilization (through utilization variable z_t), habits in consumption (through parameter h) and ad-hoc real wage rigidities (governed by parameter ρ_w^k), as well as specific AR(1) processes for the discount factor and cost-push shock processes, Ω_t and Ξ_t. The general model nests the simple special case of the main text as follows:

- complete markets: replacing (D.23) and (D.24) with the perfect risk sharing condition $\frac{\lambda_t^k}{\lambda_t^E} = \frac{\overline{\lambda}^k}{\overline{\lambda}^E}$ for $k \in (H, L)$, and also leaving out bond market clearing (D.56) and individual budget constraints (D.19), (D.20) (except for the steady state calculation where the risk sharing conditions vanish, so budget constraints are still required to pin down the steady-state consumption distribution)
  - (financial autarky: replacing (D.23) and (D.24) with $b_t^H = b_t^L = 0$)
- non-varying capital utilization: replacing (D.16) with $z_t = 1$
- no habits in consumption: set $h = 0$
- no real wage rigidities: set $\rho_w^H = \rho_w^L = 0$
- no discount factor shocks: replacing (D.12) with $\beta_t = \beta$ and leaving (D.57)
- no cost-push shocks: leaving Ξ_t from (D.41) and leaving (D.61)
- benchmark Cobb-Douglas production function: replacing (D.32) with $Y_t = A_t K_t^\iota \left[\varpi (N_t^H)^\upsilon + (1-\varpi)(N_t^L)^\upsilon\right]^{\frac{1-\iota}{\upsilon}}$ and modifying (D.33), (D.34) and (D.35) accordingly.

Variables

- search and matching: $m_t^H, m_t^L, \theta_t^H, \theta_t^L, \nu_t^H, \nu_t^L, \mu_t^H, \mu_t^L$ (8)
- household: $l_t^H, l_t^L, n_t^H, n_t^L, u_t^H, u_t^L, i_t, k_t, \beta_t, z_t$ (10)
  $c_t^E, c_t^H, c_t^L, b_t^E, b_t^H, b_t^L$ (6)
  $\lambda_t^{c,E}, \lambda_t^{c,H}, \lambda_t^{c,L}, \lambda_t^{n,H}, \lambda_t^{n,L}$ (5)
- intermediate firms: $Y_t, v_t^H, v_t^L, K_t, F_{N,t}^H, F_{N,t}^L, F_{K,t}$ (7)
- prices: $R_t, r_t, w_t^H, w_t^L, w_t^{H*}, w_t^{L*}, x_t, \Pi_t, \Delta_t, \Theta_t$ (10)
- government: t_t^E, t_t^H, t_t^L (3)
- aggregate variables: $N_t^H, N_t^L, U_t^H, U_t^L, C_t, I_t, T_t$ (7)
- other labor market measures: $partic_t^H, partic_t^L, unemp_t^H, unemp_t^L, bw_t^H, bw_t^L$ (6)
- exogenous processes: $\Omega_t, A_t, G_t, e_t, \Xi_t$ (5)
- **TOTAL: 67**
- shocks: $\varepsilon_t^\Omega, \varepsilon_t^a, \varepsilon_t^g, \varepsilon_t^R$ (4)

Equations .

Search and matching on the labor market:

$$m_t^H = \psi^H \left(v_t^H\right)^\varsigma \left(U_t^H\right)^{1-\varsigma} \tag{D.1}$$
$$m_t^L = \psi^L \left(v_t^L\right)^\varsigma \left(U_t^L\right)^{1-\varsigma} \tag{D.2}$$
$$\theta_t^H = \frac{v_t^H}{U_t^H} \tag{D.3}$$
$$\theta_t^L = \frac{v_t^L}{U_t^L} \tag{D.4}$$
$$\nu_t^H = \frac{m_t^H}{v_t^H} \tag{D.5}$$
$$\nu_t^L = \frac{m_t^L}{v_t^L} \tag{D.6}$$
$$\mu_t^H = \frac{m_t^H}{U_t^H} \tag{D.7}$$
$$\mu_t^L = \frac{m_t^L}{U_t^L} \tag{D.8}$$
$$N_{t+1}^H = (1-\sigma^H)N_t^H + m_t^H \tag{D.9}$$
$$N_{t+1}^L = (1-\sigma^L)N_t^L + m_t^L \tag{D.10}$$

Entrepreneurs:

$$i_t = k_{t+1} - \left(1 - \delta z_t^{\phi_z}\right)k_t + \frac{\omega}{2}\left(\frac{k_{t+1}}{k_t} - 1\right)^2 k_t \tag{D.11}$$
$$\beta_t = \beta\, \Omega_t \tag{D.12}$$
$$(c_t^E - h\, c_{t-1}^E)^{-\eta} = \lambda_t^{c,E} \tag{D.13}$$
$$\lambda_t^{c,E}\left[1 + \omega\left(\frac{k_{t+1}}{k_t} - 1\right)\right] = \beta_t E_t \lambda_{t+1}^{c,E}\left\{1 + r_{t+1}z_{t+1} - \delta(z_{t+1}) + \frac{\omega}{2}\left[\left(\frac{k_{t+2}}{k_{t+1}}\right)^2 - 1\right]\right\} \tag{D.14}$$
$$\lambda_t^{c,E} = \beta_t E_t \lambda_{t+1}^{c,E}\frac{R_t}{\Pi_{t+1}} \tag{D.15}$$
$$r_t = \delta'(z_t) = \delta\phi_z\, z_t^{\phi_z - 1} \tag{D.16}$$

Workers:

$$1 = n_t^H + u_t^H + l_t^H \tag{D.17}$$
$$1 = n_t^L + u_t^L + l_t^L \tag{D.18}$$
$$c_t^H + t_t^H + b_t^H = w_t^H n_t^H + \frac{R_{t-1}}{\Pi_t}b_{t-1}^H + \varkappa^H u_t^H \tag{D.19}$$
$$c_t^L + t_t^L + b_t^L = w_t^L n_t^L + \frac{R_{t-1}}{\Pi_t}b_{t-1}^L + \varkappa^L u_t^L \tag{D.20}$$
$$(c_t^H - h\, c_{t-1}^H)^{-\eta} = \lambda_t^{c,H} \tag{D.21}$$
$$(c_t^L - h\, c_{t-1}^L)^{-\eta} = \lambda_t^{c,L} \tag{D.22}$$
$$\lambda_t^{c,H}\left[1 + \xi_b\left(b_t^H - \bar{b}^H\right)\right] = \beta_t E_t \lambda_{t+1}^{c,H}\frac{R_t}{\Pi_{t+1}} \tag{D.23}$$
$$\lambda_t^{c,L}\left[1 + \xi_b\left(b_t^L - \bar{b}^L\right)\right] = \beta_t E_t \lambda_{t+1}^{c,L}\frac{R_t}{\Pi_{t+1}} \tag{D.24}$$
$$\lambda_t^{n,H} = \frac{\Phi^H \left(l_t^H\right)^{-\xi} - \varkappa^H \lambda_t^{c,H}}{\mu_t^H} \tag{D.25}$$
$$\lambda_t^{n,L} = \frac{\Phi^L \left(l_t^L\right)^{-\xi} - \varkappa^L \lambda_t^{c,L}}{\mu_t^L} \tag{D.26}$$
$$\lambda_t^{n,H} = \beta_t E_t\left[\lambda_{t+1}^{c,H}w_{t+1}^H + (1-\sigma^H)\lambda_{t+1}^{n,H} - \Phi^H\left(l_{t+1}^H\right)^{-\xi}\right] \tag{D.27}$$
$$\lambda_t^{n,L} = \beta_t E_t\left[\lambda_{t+1}^{c,L}w_{t+1}^L + (1-\sigma^L)\lambda_{t+1}^{n,L} - \Phi^L\left(l_{t+1}^L\right)^{-\xi}\right] \tag{D.28}$$

Intermediate firm:

$$r_t = x_t F_{K,t} \tag{D.29}$$

$$\frac{\kappa^H}{\nu_t^H} = \beta_t E_t \frac{\lambda_{t+1}^{c,E}}{\lambda_t^{c,E}} \left[x_{t+1} F_{N,t+1}^H - w_{t+1}^H + (1-\sigma^H) \frac{\kappa^H}{\nu_{t+1}^H} \right] \tag{D.30}$$

$$\frac{\kappa^L}{\nu_t^L} = \beta_t E_t \frac{\lambda_{t+1}^{c,E}}{\lambda_t^{c,E}} \left[x_{t+1} F_{N,t+1}^L - w_{t+1}^L + (1-\sigma^L) \frac{\kappa^L}{\nu_{t+1}^L} \right] \tag{D.31}$$

$$Y_t = A_t \left[\phi \left[\lambda\, K_t^\gamma + (1-\lambda)(N_t^H)^\gamma \right]^{\frac{\alpha}{\gamma}} + (1-\phi)(N_t^L)^\alpha \right]^{\frac{1}{\alpha}} \tag{D.32}$$

$$F_{N,t}^H = \phi(1-\lambda) A_t \left[\phi \left[\lambda\, K_t^\gamma + (1-\lambda)(N_t^H)^\gamma \right]^{\frac{\alpha}{\gamma}} + (1-\phi)(N_t^L)^\alpha \right]^{\frac{1-\alpha}{\alpha}} \left[\lambda\, K_t^\gamma + (1-\lambda)(N_t^H)^\gamma \right]^{\frac{\alpha-\gamma}{\gamma}} (N_t^H)^{\gamma-1} \tag{D.33}$$

$$F_{K,t} = \phi\lambda\, A_t \left[\phi \left[\lambda\, K_t^\gamma + (1-\lambda)(N_t^H)^\gamma \right]^{\frac{\alpha}{\gamma}} + (1-\phi)(N_t^L)^\alpha \right]^{\frac{1-\alpha}{\alpha}} \left[\lambda\, K_t^\gamma + (1-\lambda)(N_t^H)^\gamma \right]^{\frac{\alpha-\gamma}{\gamma}} (K_t)^{\gamma-1} \tag{D.34}$$

$$F_{N,t}^L = (1-\phi) A_t \left[\phi \left[\lambda\, K_t^\gamma + (1-\lambda)(N_t^H)^\gamma \right]^{\frac{\alpha}{\gamma}} + (1-\phi)(N_t^L)^\alpha \right]^{\frac{1-\alpha}{\alpha}} (N_t^L)^{\alpha-1} \tag{D.35}$$

Wage-bargaining:

$$w_t^{H*} = \vartheta^H \left[x_t F_{N,t}^H + (1-\sigma^H) \frac{\kappa^H}{\nu_t^H} \right] + \frac{1-\vartheta^H}{\lambda_t^{c,H}} \left[\Phi^H \left(l_t^H \right)^{-\xi} - (1-\sigma^H) \lambda_t^{n,H} \right] \tag{D.36}$$

$$w_t^{L*} = \vartheta^L \left[x_t F_{N,t}^L + (1-\sigma^L) \frac{\kappa^L}{\nu_t^L} \right] + \frac{1-\vartheta^L}{\lambda_t^{c,L}} \left[\Phi^L \left(l_t^L \right)^{-\xi} - (1-\sigma^L) \lambda_t^{n,L} \right] \tag{D.37}$$

$$w_t^H = \rho_w^H w_{t-1}^H + (1-\rho_w^H) w_t^{H*} \tag{D.38}$$

$$w_t^L = \rho_w^L w_{t-1}^L + (1-\rho_w^L) w_t^{L*} \tag{D.39}$$

Retail firms – NKPC:

$$\frac{\Theta_t}{\Delta_t} = \left[\frac{1-\chi \Pi_t^{\epsilon-1}}{1-\chi} \right]^{\frac{1}{1-\epsilon}} \tag{D.40}$$

$$\Theta_t = Y_t (c_t^E - h\, c_{t-1}^E)^{-\eta}\ x_t\ \Xi_t\ \frac{(1-\tau)\epsilon}{\epsilon-1}\ +\ \chi\ \beta_t E_t \left[\Pi_{t+1}^\epsilon \Theta_{t+1} \right] \tag{D.41}$$

$$\Delta_t = Y_t (c_t^E - h\, c_{t-1}^E)^{-\eta}\ +\ \chi\ \beta_t E_t \left[\Pi_{t+1}^{\epsilon-1} \Delta_{t+1} \right] \tag{D.42}$$

Government policies:

$$T_t = \varkappa^H U_t^H + \varkappa^L U_t^L + G_t + \tau x_t Y_t \tag{D.43}$$

$$T_t = \varphi^E t_t^E + \varphi^H t_t^H + \varphi^L t_t^L \tag{D.44}$$

$$t_t^H = T_t \tag{D.45}$$

$$t_t^L = T_t \tag{D.46}$$

$$\frac{R_t}{\overline{R}} = \left(\frac{R_{t-1}}{\overline{R}} \right)^{\zeta^R} \left[\left(\frac{\Pi_t}{\overline{\Pi}} \right)^{\zeta^\pi} \left(\frac{U_t^H + U_t^L}{\overline{U}^H + \overline{U}^L} \right)^{\zeta^u} e_t \right]^{1-\zeta^R} \tag{D.47}$$

Aggregate variables and market clearings:

$$C_t = \varphi^E c_t^E + \varphi^H c_t^H + \varphi^L c_t^L \tag{D.48}$$

$$I_t = \varphi^E i_t \tag{D.49}$$

$$U_t^H = \varphi^H u_t^H \tag{D.50}$$

$$U_t^L = \varphi^L u_t^L \tag{D.51}$$

$$N_t^H = \varphi^H n_t^H \tag{D.52}$$

$$N_t^L = \varphi^L n_t^L \tag{D.53}$$

$$K_t = \varphi^E z_t k_t \tag{D.54}$$

$$Y_t = C_t + I_t + G_t + \kappa^H v_t^H + \kappa^L v_t^L \tag{D.55}$$

$$0 = \varphi^E b_t^E + \varphi^H b_t^H + \varphi^L b_t^L \tag{D.56}$$

Exogenous processes:

$$\Omega_t = (\Omega_{t-1})^{\rho_\Omega} \exp(\varepsilon_t^\Omega) \tag{D.57}$$

$$A_t = (A_{t-1})^{\rho_a} \exp(\varepsilon_t^a) \tag{D.58}$$

$$G_t = (\overline{y}\Gamma)^{1-\rho_g} (G_{t-1})^{\rho_g} \exp(\varepsilon_t^g) \tag{D.59}$$

$$e_t = (e_{t-1})^{\rho_R} \exp(\varepsilon_t^R) \tag{D.60}$$

$$\Xi_t = (\Xi_{t-1})^{\rho_\Xi} \exp(\varepsilon_t^\Xi) \tag{D.61}$$

Other definitions:

$$partic_t^H \equiv \frac{N_t^H + U_t^H}{\varphi^H} \tag{D.62}$$

$$partic_t^L \equiv \frac{N_t^L + U_t^L}{\varphi^L} \tag{D.63}$$

$$unemp_t^H \equiv \frac{U_t^H}{N_t^H + U_t^H} \tag{D.64}$$

$$unemp_t^L \equiv \frac{U_t^L}{N_t^L + U_t^L} \tag{D.65}$$

$$bw_t^H \equiv \frac{\varkappa^H}{w_t^H} \tag{D.66}$$

$$bw_t^L \equiv \frac{\varkappa^L}{w_t^L} \tag{D.67}$$

$$partic_t = 1 - \varphi^H l_t^H - \varphi^L l_t^L$$

$$unemp_t = \frac{U_t^H + U_t^L}{partic_t}$$

$$\theta_t = \frac{v_t^H + v_t^L}{U_t^H + U_t^L}$$

$$wageprem_t = \frac{w_t^H}{w_t^L}$$

$$\Psi_{H,L} = \frac{w_t^H N_t^H}{w_t^L N_t^L}$$

D.2 Steady state

This is a recursion which allows to calculate the steady state of the model step-by step, without having to solve a huge non-linear system of equations. Left-hand side equation numbers refer back to which original equilibrium condition was used.

Labor market SAM .

- We have exogenized 4 targeted steady state values (in blue) which implies the need to endogenize 4 more parameters outside this subsystem later on ($\vartheta^H, \vartheta^L, \Phi^H, \Phi^L$).

(D.62) $$U^H = \varphi^H \, partic^H \, unemp^H \tag{D.68}$$

(D.63) $$U^L = \varphi^L \, partic^L \, unemp^L \tag{D.69}$$

(D.64) $$N^H = \left[\frac{1}{unemp^H} - 1\right] U^H \tag{D.70}$$

(D.65) $$N^L = \left[\frac{1}{unemp^L} - 1\right] U^L \tag{D.71}$$

(D.9) $$m^H = \sigma^H N^H \tag{D.72}$$

(D.10) $$m^L = \sigma^L N^L \tag{D.73}$$

(D.1) $$v^H = \left[\frac{m^H}{\psi^H (U^H)^{1-\varsigma}}\right]^{\frac{1}{\varsigma}} \tag{D.74}$$

(D.2) $$v^L = \left[\frac{m^L}{\psi^L (U^L)^{1-\varsigma}}\right]^{\frac{1}{\varsigma}} \tag{D.75}$$

(D.7) $$\mu^H = \frac{m^H}{U^H} \tag{D.76}$$

(D.8) $$\mu^L = \frac{m^L}{U^L} \tag{D.77}$$

(D.5) $$\nu^H = \frac{m^H}{v^H} \tag{D.78}$$

(D.6) $$\nu^L = \frac{m^L}{v^L} \tag{D.79}$$

(D.3) $$\theta^H = \frac{v^H}{U^H} \tag{D.80}$$

(D.4) $$\theta^L = \frac{v^L}{U^L} \tag{D.81}$$

(D.50) $$u^H = \frac{U^H}{\varphi^H} \tag{D.82}$$

(D.51) $$u^L = \frac{U^L}{\varphi^L} \tag{D.83}$$

(D.52) $$n^H = \frac{N^H}{\varphi^H} \tag{D.84}$$

(D.52) $$n^L = \frac{N^L}{\varphi^L} \tag{D.85}$$

Households .

(D.17) $$l^H = 1 - n^H - u^H \tag{D.86}$$

(D.18) $$l^L = 1 - n^L - u^L \tag{D.87}$$

Capital utilization in steady state is exogeneously set equal to 1, by endogenizing the parameter ϕ_z

$$z = 1$$

$$\phi_z$$

$$\text{(D.57)} \qquad \Omega = 1 \tag{D.88}$$

$$\text{(D.12)} \qquad \beta = \beta\Omega = \beta \tag{D.89}$$

$$\text{(D.14)} \qquad 1 = \beta\left[1 - \delta + r\right]$$

$$r = \frac{1-\beta}{\beta} + \delta \tag{D.90}$$

$$\text{(D.16)} \qquad \phi_z = \frac{r}{\delta} = 1 + \frac{1-\beta}{\delta\beta} \tag{D.91}$$

Pricing and bonds .

Steady-state inflation rate is determined by the central bank's target $\overline{\Pi}$ which is set equal to 1. We endogenize real marginal costs x to be 1 in steady state (i.e. to offset the static distortion coming from monopolistic competition), by endogenizing employment subsidy τ.

$$x = 1$$

$$\tau$$

$$\text{(D.60)} \qquad e = 1 \tag{D.92}$$

$$\text{(D.47)} \qquad \Pi = \overline{\Pi}/e = \overline{\Pi} = 1 \tag{D.93}$$

$$\text{(D.15)} \qquad R = \frac{\Pi}{\beta} = \frac{1}{\beta} \tag{D.94}$$

$$\text{(D.23)} \qquad b^{H*} = \bar{b}^H + \left[\beta\frac{R}{\Pi} - 1\right]/\xi_b = \bar{b}^H \tag{D.95}$$

$$\text{(D.24)} \qquad b^{L*} = \bar{b}^L + \left[\beta\frac{R}{\Pi} - 1\right]/\xi_b = \bar{b}^L \tag{D.96}$$

$$\text{(D.40)} \qquad x = \frac{\epsilon - 1}{\epsilon}\frac{1}{1-\tau} = 1$$

$$\tau = \frac{1}{\epsilon} \tag{D.97}$$

Output and Wages .

(D.58) $$A = 1 \tag{D.98}$$

(D.29) $$F_K = \frac{r}{x} = r \tag{D.99}$$

(D.34) $$K = F_K^{-1}(F_K, N^H, N^L) \quad \texttt{by numerical solver only - no symbolic} \tag{D.100}$$

(D.54) $$k = \frac{K}{z\varphi^E} \tag{D.101}$$

(D.32) $$Y = A\left[\phi\left[\lambda\, K^\gamma + (1-\lambda)(N^H)^\gamma\right]^{\frac{\alpha}{\gamma}} + (1-\phi)(N^L)^\alpha\right]^{\frac{1}{\alpha}} \tag{D.102}$$

(D.33) $$F_N^H = \phi(1-\lambda)A\left[\phi\left[\lambda\, K^\gamma + (1-\lambda)(N^H)^\gamma\right]^{\frac{\alpha}{\gamma}} + (1-\phi)(N^L)^\alpha\right]^{\frac{1-\alpha}{\alpha}} \left[\lambda\, K^\gamma + (1-\lambda)(N^H)^\gamma\right]^{\frac{\alpha-\gamma}{\gamma}} (N^H)^{\gamma-1} \tag{D.103}$$

(D.35) $$F_N^L = (1-\phi)A\left[\phi\left[\lambda\, K^\gamma + (1-\lambda)(N^H)^\gamma\right]^{\frac{\alpha}{\gamma}} + (1-\phi)(N^L)^\alpha\right]^{\frac{1-\alpha}{\alpha}} (N^L)^{\alpha-1} \tag{D.104}$$

(D.30) $$w^H = x\, F_N^H + \left(1 - \sigma^H - \frac{1}{\beta}\right)\frac{\kappa^H}{\nu^H} \tag{D.105}$$

(D.31) $$w^L = x\, F_N^L + \left(1 - \sigma^L - \frac{1}{\beta}\right)\frac{\kappa^L}{\nu^L} \tag{D.106}$$

Replacement rates of unemployment benefits are targeted exogenously, by endogenizing benefits in levels.

$$bw^H, bw^L$$
$$\varkappa^H, \varkappa^L$$

(D.66) $$\varkappa^H = bw^H w^H \tag{D.107}$$

(D.67) $$\varkappa^L = bw^L w^L \tag{D.108}$$

Market clearing .

(D.59) $$G = Y\,\Gamma \tag{D.109}$$

(D.43) $$T = \varkappa^H U^H + \varkappa^L U^L + G + \tau x y \tag{D.110}$$

(D.45) $$t^H = \mathcal{T}^H = T \quad \text{arbitrary (equal) tax distribution} \tag{D.111}$$

(D.46) $$t^L = \mathcal{T}^L = T \quad \text{arbitrary (equal) tax distribution} \tag{D.112}$$

(D.44) $$t^E = \frac{T - \varphi^H t^H - \varphi^L t^L}{\varphi^E} \tag{D.113}$$

(D.11) $$i = \delta k \tag{D.114}$$

(D.49) $$I = \varphi^E i \tag{D.115}$$

(D.56) $$b^{E*} = \frac{0 - \varphi^H b^{H*} - \varphi^L b^{L*}}{\varphi^E} \tag{D.116}$$

(D.55) $$C = Y - I - G - \kappa^H v^H - \kappa^L v^L \tag{D.117}$$

(D.19) $$c^H = w^H n^H + \varkappa^H u^H - t^H - b^{H*}\left(1 - \frac{R}{\Pi}\right) \tag{D.118}$$

(D.20) $$c^L = w^L n^L + \varkappa^L u^L - t^L - b^{L*}\left(1 - \frac{R}{\Pi}\right) \tag{D.119}$$

(D.48) $$c^E = \frac{C - \varphi^H c^H - \varphi^L c^L}{\varphi^E} \tag{D.120}$$

Parameters and Lagrange-multipliers .

$$\text{(D.21)} \qquad \lambda^{c,H} = [(1-h)c^H]^{-\eta} \tag{D.121}$$

$$\text{(D.22)} \qquad \lambda^{c,L} = [(1-h)c^L]^{-\eta} \tag{D.122}$$

$$\text{(D.13)} \qquad \lambda^{c,E} = [(1-h)c^E]^{-\eta} \tag{D.123}$$

$$\text{(D.61)} \qquad \Xi = 1 \tag{D.124}$$

$$\text{(D.41)} \qquad \Theta = \frac{Y\,[(1-h)c^E]^{-\eta}\,x\,\Xi\,\frac{(1-\tau)\epsilon}{\epsilon-1}}{1-\chi\beta} \tag{D.125}$$

$$\text{(D.42)} \qquad \Delta = \frac{Y\,[(1-h)c^E]^{-\eta}}{1-\chi\beta} \tag{D.126}$$

Now we take the following 2×2 equations (D.25)-(D.28) and solve them as two separate systems for $\Phi^k, \lambda^{n,k} \quad k \in H, L$.[2]

$$\text{(D.25)} \qquad \lambda^{n,H} = \frac{\Phi^H \left(l_t^H\right)^{-\xi} - \varkappa^H \lambda^{c,H}}{\mu^H} \tag{D.127}$$

$$\text{(D.27)} \qquad \lambda^{n,H} = E_t\beta\left[\lambda^{c,H}w^H + (1-\sigma^H)\lambda^{n,H} - \Phi^H \left(l^H\right)^{-\xi}\right] \tag{D.128}$$

$$\text{(D.26)} \qquad \lambda^{n,L} = \frac{\Phi^L \left(l_t^L\right)^{-\xi} - \varkappa^L \lambda^{c,L}}{\mu^L} \tag{D.129}$$

$$\text{(D.28)} \qquad \lambda^{n,L} = E_t\beta\left[\lambda^{c,L}w^L + (1-\sigma^L)\lambda^{n,L} - \Phi^L \left(l^L\right)^{-\xi}\right] \tag{D.130}$$

This can be solved by the symbolic toolbox of Matlab.

In a similar way we recover the remaining 2 endogenized parameters ϑ^H, ϑ^L by solving these equations in the symbolic toolbox:

$$\text{(D.36)} \qquad w^{H*} = \vartheta^H\left[xF_N^H + (1-\sigma^H)\frac{\kappa^H}{\nu^H}\right] + \frac{1-\vartheta^H}{\lambda^{c,H}}\left[\Phi^H \left(l^H\right)^{-\xi} - (1-\sigma^H)\lambda^{n,H}\right] \tag{D.131}$$

$$\text{(D.37)} \qquad w^{L*} = \vartheta^L\left[xF_N^L + (1-\sigma^L)\frac{\kappa^L}{\nu^L}\right] + \frac{1-\vartheta^L}{\lambda^{c,L}}\left[\Phi^L \left(l^L\right)^{-\xi} - (1-\sigma^L)\lambda^{n,L}\right] \tag{D.132}$$

$$\text{(D.38)} \qquad w^H = w^{H*} \tag{D.133}$$

$$\text{(D.39)} \qquad w^L = w^{L*} \tag{D.134}$$

We have checked that ϑ^k is between 0 and 1.

With this, the steady-state values of all 65 variables are computed.

- exogenous (targeted) steady states, 8: $partic^H$, $unemp^H$, $partic^L$, $unemp^L$, bw^H, bw^L, z, x
- endogenous parameters, 8: $\Phi^H, \Phi^L, \vartheta^H, \vartheta^L, \phi_z, \tau, \varkappa^H, \varkappa^L$

[2] Note that Φ^k are 2 parameters to endogenize out of those 4 which we were left with after the SAM section. The remaining 2 parameter are: ϑ^k.

E Wage dynamics decomposition

E.1 Log-linear dynamic equations

E.1.1 Wage bargaining equation

Taking dynamic equilibrium conditions (D.36)-(D.37), i.e. the wage bargaining equations become:

$$
\begin{aligned}
w_t^k &= \vartheta^k \left[x_t F_{N,t}^k + (1-\sigma^k) \frac{\kappa^k}{\nu_t^k} \right] + \frac{1-\vartheta^k}{\lambda_t^{c,k}} \left[\Phi^k \left(l_t^k \right)^{-\xi} - (1-\sigma^k) \lambda_t^{n,k} \right] \equiv \\
\equiv \quad g(w_t^k) &= f\left(x_t, F_{N,t}^k, \nu_t^k, \lambda_t^{c,k}, l_t^k, \lambda_t^{n,k} \right)
\end{aligned}
$$

Log-linearization of the above expression yields:

$$
\begin{aligned}
\underbrace{1}_{g_w(w^k)} \; w^k \; \widehat{w}_t^k = & \underbrace{\vartheta^k F_N^k}_{f_x\left(x, F_N^k, \nu^k, \lambda^{c,k}, l^k, \lambda^{n,k}\right)} \; x \; \widehat{x}_t \; + \\
& + \underbrace{\vartheta^k x}_{f_{F_N^k}\left(x, F_N^k, \nu^k, \lambda^{c,k}, l^k, \lambda^{n,k}\right)} \; F_N^k \; \widehat{F}_{N,t}^k \; + \\
& + \underbrace{(-\vartheta^k)(1-\sigma^k) \frac{\kappa^k}{(\nu^k)^2}}_{f_{\nu^k}\left(x, F_N^k, \nu^k, \lambda^{c,k}, l^k, \lambda^{n,k}\right)} \; \nu^k \; \widehat{\nu}_t^k \; + \\
& + \underbrace{\left(-\frac{1-\vartheta^k}{(\lambda^{c,k})^2} \right) \left[\Phi^k (l^k)^{-\xi} - (1-\sigma^k) \lambda^{n,k} \right]}_{f_{\lambda^{c,k}}\left(x, F_N^k, \nu^k, \lambda^{c,k}, l^k, \lambda^{n,k}\right)} \; \lambda^{c,k} \; \widehat{\lambda}_t^{c,k} \; + \\
& + \underbrace{\frac{1-\vartheta^k}{\lambda^{c,k}} \left[-\xi \; \Phi^k (l^k)^{-\xi-1} \right]}_{f_{l^k}\left(x, F_N^k, \nu^k, \lambda^{c,k}, l^k, \lambda^{n,k}\right)} \; l^k \; \widehat{l}_t^k \; + \\
& + \underbrace{\left(-\frac{1-\vartheta^k}{\lambda^{c,k}} \right) (1-\sigma^k)}_{f_{\lambda^{n,k}}\left(x, F_N^k, \nu^k, \lambda^{c,k}, l^k, \lambda^{n,k}\right)} \; \lambda^{n,k} \; \widehat{\lambda}_t^{n,k}
\end{aligned}
$$

Rearranging terms we get:

$$
\begin{aligned}
\widehat{w}_t^k = & \frac{\vartheta^k x F_N^k}{w^k} \; \widehat{x}_t \; + \; \frac{\vartheta^k x F_N^k}{w^k} \; \widehat{F}_{N,t}^k \; - \; \frac{\vartheta^k (1-\sigma^k) \kappa^k}{\nu^k w^k} \; \widehat{\nu}_t^k \; - \\
& - \frac{1-\vartheta^k}{\lambda^{c,k} w^k} \left[\Phi^k (l^k)^{-\xi} - (1-\sigma^k) \lambda^{n,k} \right] \widehat{\lambda}_t^{c,k} \; - \\
& - \xi \; \frac{(1-\vartheta^k) \; \Phi^k (l^k)^{-\xi}}{\lambda^{c,k} \; w^k} \; \widehat{l}_t^k \; - \; \frac{(1-\vartheta^k)(1-\sigma^k) \lambda^{n,k}}{\lambda^{c,k} \; w^k} \; \widehat{\lambda}_t^{n,k}
\end{aligned} \tag{E.1}
$$

where the blue terms are related with the dynamics of the firm's surplus and labor *demand*, while the red terms represent movements in the worker's surplus and labor *supply*.

E.1.2 Marginal utility of consumption

Based on (D.21)-(D.22), and setting $h = 0$

$$
\lambda_t^{c,k} = (c_t^k)^{-\eta}
$$

the log-linear approximation becomes

$$\lambda^{c,k}\,\widehat{\lambda}_t^{c,k} = -\eta\,\underbrace{(c^k)^{-\eta-1}\,c^k}_{\lambda^{c,k}}\,\widehat{c}_t^k$$

$$\widehat{\lambda}_t^{c,k} = -\eta\,\widehat{c}_t^k \tag{E.2}$$

E.1.3 Vacancy filling rate

Based on (D.5)-(D.6), combined with the matching functions in (D.1)-(D.2) and the definition of labor market tightness θ^k in (D.3)-(D.4):

$$\nu_t^k = \frac{m_t^k}{v_t^k} = \frac{\psi^k (v_t^k)^{\varsigma}(U_t^k)^{1-\varsigma}}{v_t^k} = \psi^k \left(\frac{v_t^k}{U_t^k}\right)^{\varsigma-1} = \\ = \psi^k \left(\theta_t^k\right)^{\varsigma-1}$$

the log-linear approximation becomes:

$$\nu^k\,\widehat{\nu}_t^k = (\varsigma-1)\,\underbrace{\psi^k(\theta^k)^{\varsigma-2}\,\theta^k}_{\nu^k}\,\widehat{\theta}_t^k$$

$$\widehat{\nu}_t^k = (\varsigma-1)\,\widehat{\theta}_t^k \tag{E.3}$$

E.1.4 Job finding rate

Based on (D.7)-(D.8), combined with the matching functions in (D.1)-(D.2) and the definition of tightness θ^k in (D.3)-(D.4):

$$\mu_t^k = \frac{m_t^k}{U_t^k} = \frac{\psi^k (v_t^k)^{\varsigma}(U_t^k)^{1-\varsigma}}{U_t^k} = \psi^k \left(\frac{v_t^k}{U_t^k}\right)^{\varsigma} = \\ = \psi^k \left(\theta_t^k\right)^{\varsigma}$$

the log-linear approximation becomes

$$\mu^k\,\widehat{\mu}_t^k = (\varsigma)\,\underbrace{\psi^k(\theta^k)^{\varsigma-1}\,\theta^k}_{\mu^k}\,\widehat{\theta}_t^k$$

$$\widehat{\mu}_t^k = \varsigma\,\widehat{\theta}_t^k \tag{E.4}$$

E.1.5 Participation choice – household FOC

To express the Lagrange-multiplier $\lambda_t^{n,k}$ in the SAM-constraint, we turn to the household FOC w.r.t. u_t^k describing the participation (labor supply) decision, (D.25)-(D.26):

$$\lambda_t^{n,k} = \frac{\Phi^k\left(l_t^k\right)^{-\xi} - \varkappa^k \lambda_t^{c,k}}{\mu_t^k}$$

The log-linear approximation then becomes:

$$\lambda^{n,k}\widehat{\lambda}_t^{n,k} = -\xi\,\frac{\Phi^k(l^k)^{-\xi-1}}{\mu^k}\,l^k\,\widehat{l}_t^k \;-\; \underbrace{\frac{\Phi^k(l^k)^{-\xi} - \varkappa^k\lambda^{c,k}}{(\mu^k)^2}\,\mu^k}_{\lambda^{n,k}}\,\widehat{\mu}_t^k \;-\; \frac{\varkappa^k}{\mu^k}\,\lambda^{c,k}\,\widehat{\lambda}_t^{c,k}$$

$$\widehat{\lambda}_t^{n,k} = -\xi\,\frac{\Phi^k(l^k)^{-\xi}}{\mu^k\,\lambda^{n,k}}\,\widehat{l}_t^k \;-\; \widehat{\mu}_t^k \;-\; \frac{\varkappa^k\,\lambda^{c,k}}{\mu^k\,\lambda^{n,k}}\,\widehat{\lambda}_t^{c,k}$$

in which we can further insert our previous results (E.2) and (E.4):

$$\widehat{\lambda}_t^{n,k} = -\xi\,\frac{\Phi^k(l^k)^{-\xi}}{\mu^k\,\lambda^{n,k}}\,\widehat{l}_t^k \;-\; \varsigma\,\widehat{\theta}_t^k \;+\; \eta\,\frac{\varkappa^k\,\lambda^{c,k}}{\mu^k\,\lambda^{n,k}}\,\widehat{c}_t^k \tag{E.5}$$

E.1.6 Wage bargaining – substituted in

In order to facilitate interpretation of (E.1), we substitute out the Lagrange multipliers and the vacancy filling rate by using (E.2), (E.3) and (E.5).

$$\begin{aligned}
\widehat{w}_t^k &= \frac{\vartheta^k x F_N^k}{w^k}\,\widehat{x}_t + \frac{\vartheta^k x F_N^k}{w^k}\,\widehat{F}_{N,t}^k - \frac{\vartheta^k(1-\sigma^k)\kappa^k}{\nu^k w^k}\,\widehat{\nu}_t^k - \\
&- \frac{1-\vartheta^k}{\lambda^{c,k}w^k}\left[\Phi^k(l^k)^{-\xi} - (1-\sigma^k)\lambda^{n,k}\right]\widehat{\lambda}_t^{c,k} - \xi\,\frac{(1-\vartheta^k)\,\Phi^k(l^k)^{-\xi}}{\lambda^{c,k}\,w^k}\,\widehat{l}_t^k - \\
&- \frac{(1-\vartheta^k)(1-\sigma^k)\lambda^{n,k}}{\lambda^{c,k}\,w^k}\,\widehat{\lambda}_t^{n,k} = \\
&= \frac{\vartheta^k x F_N^k}{w^k}\,\widehat{x}_t + \frac{\vartheta^k x F_N^k}{w^k}\,\widehat{F}_{N,t}^k + (1-\varsigma)\frac{\vartheta^k(1-\sigma^k)\kappa^k}{\nu^k w^k}\,\widehat{\theta}_t^k + \\
&+ \eta\,\frac{1-\vartheta^k}{\lambda^{c,k}w^k}\left[\Phi^k(l^k)^{-\xi} - (1-\sigma^k)\lambda^{n,k}\right]\widehat{c}_t^k - \xi\,\frac{(1-\vartheta^k)\,\Phi^k(l^k)^{-\xi}}{\lambda^{c,k}\,w^k}\,\widehat{l}_t^k + \\
&+ \frac{(1-\vartheta^k)(1-\sigma^k)\lambda^{n,k}}{\lambda^{c,k}\,w^k}\left[\xi\,\frac{\Phi^k(l^k)^{-\xi}}{\mu^k\,\lambda^{n,k}}\,\widehat{l}_t^k + \varsigma\,\widehat{\theta}_t^k - \eta\,\frac{\varkappa^k\,\lambda^{c,k}}{\mu^k\,\lambda^{n,k}}\,\widehat{c}_t^k\right]
\end{aligned}$$

$$\begin{aligned}
\widehat{w}_t^k &= \frac{\vartheta^k x F_N^k}{w^k}\,\widehat{x}_t + \frac{\vartheta^k x F_N^k}{w^k}\,\widehat{F}_{N,t}^k + \\
&+ \frac{1-\sigma^k}{w^k}\left[(1-\varsigma)\frac{\vartheta^k\kappa^k}{\nu^k} + \varsigma\frac{(1-\vartheta^k)\lambda^{n,k}}{\lambda^{c,k}}\right]\widehat{\theta}_t^k + \\
&+ \frac{\eta(1-\vartheta^k)}{w^k}\left[\frac{\Phi^k(l^k)^{-\xi}}{\lambda^{c,k}} - (1-\sigma^k)\left(\frac{\lambda^{n,k}}{\lambda^{c^k}} + \frac{\varkappa^k}{\mu^k}\right)\right]\widehat{c}_t^k + \\
&+ \xi\,\frac{(1-\vartheta^k)\,\Phi^k(l^k)^{-\xi}}{\lambda^{c,k}\,w^k}\left[\frac{(1-\sigma^k)}{\mu^k} - 1\right]\widehat{l}_t^k
\end{aligned} \tag{E.6}$$

which decomposes the first-order approximation of wage dynamics into additive terms showing the contributions of dynamic changes in real marginal costs $\widehat{x}_t$ (i.e. aggregate demand pressures); marginal product of labor $\widehat{F}_{N,t}^k$; labor market tightness θ^k (both through its effect on vacancy filling $\widehat{\nu}_t^k$ and job finding $\widehat{\mu}_t^k$ rates); consumption $\widehat{c}_t^k$; and leisure $\widehat{l}_t^k$ (i.e. participation choice). As before, blue terms have to do with the firm's surplus and labor *demand*, while red terms have to do with the worker's surplus and labor *supply*.

E.2 Steady-state coefficients

In order to express the coefficients of the log-linear equation (E.6) in terms of only exogenous parameters and steady-state targets, we need to substitute out endogenous steady-state results like w^k, $\lambda^{c,k}$, $\lambda^{n,k}$, $\Phi^k(l^k)^{-\xi}$, ν^k, μ^k, ϑ^k, F_N^k.[3] In theory, this should be possible; however, a fully analytical solution is not feasible in this case, since starting from exogenous parameters and steady-state targets, solving for F_N^k has no closed-form solution, due to our complex production function (it requires a numerical solver). Therefore, the resulting coefficients will necessarily include the term F_N^k in addition to other exogenous parameters, while "behind the scenes" F_N^k is also a function of those same parameters.

[3] For a detailed description of the calculation of the steady state see Section D.2.

E.2.1 Labor market variables

Using the steady state labor market relationships (D.68)-(D.85), we can derive

$$m^k = \sigma^k \overbrace{\left[\frac{1}{unemp^k} - 1\right] \underbrace{\varphi^k\ partic^k\ unemp^k}_{U^k}}^{N^k}$$

$$\upsilon^k = \left[\frac{m^k}{\psi^k (U^k)^{1-\varsigma}}\right]^{\frac{1}{\varsigma}} = \left[\frac{\sigma^k \left[\frac{1}{unemp^k} - 1\right] (U^k)^{\varsigma}}{\psi^k}\right]^{\frac{1}{\varsigma}} =$$

$$= \left[\frac{\sigma^k}{\psi^k}\left(\frac{1}{unemp^k} - 1\right)\right]^{\frac{1}{\varsigma}} \underbrace{\varphi^k\ partic^k\ unemp^k}_{U^k}$$

and then for the steady state vacancy filling and job finding rates:

$$\mu^k = \frac{m^k}{U^k} = \sigma^k \left[\frac{1}{unemp^k} - 1\right] \tag{E.7}$$

$$\nu^k = \frac{m^k}{\upsilon^k} = \frac{\sigma^k \left[\frac{1}{unemp^k} - 1\right] U^k}{\left[\frac{\sigma^k}{\psi^k}\left(\frac{1}{unemp^k} - 1\right)\right]^{\frac{1}{\varsigma}} U^k} =$$

$$= (\psi^k)^{\frac{1}{\varsigma}} \left[\sigma^k \left(\frac{1}{unemp^k} - 1\right)\right]^{\frac{\varsigma - 1}{\varsigma}} \tag{E.8}$$

and finally

$$n^k = \frac{N^k}{\varphi^k} = \left[\frac{1}{unemp^k} - 1\right] partic^k\ unemp^k =$$

$$= (1 - unemp^k)\ partic^k \tag{E.9}$$

$$u^k = \frac{U^k}{\varphi^k} = unemp^k\ partic^k \tag{E.10}$$

E.2.2 Wage

Using the labor FOC of the firm (D.105)-(D.106), and then substituting in (E.8) we get

$$w^k = x\ F_N^k + \left(1 - \sigma^k - \frac{1}{\beta}\right)\frac{\kappa^k}{\nu^k} =$$

$$= x\ F_N^k - \kappa^k \left(\sigma^k + \frac{1-\beta}{\beta}\right)(\psi^k)^{-\frac{1}{\varsigma}} \left[\sigma^k \left(\frac{1}{unemp^k} - 1\right)\right]^{\frac{1-\varsigma}{\varsigma}} \tag{E.11}$$

where F_N^k is given by (D.103)-(D.104):

$$F_N^H = \phi(1-\lambda)A\left[\phi\left[\lambda\, K^{\gamma} + (1-\lambda)(N^H)^{\gamma}\right]^{\frac{\alpha}{\gamma}} + (1-\phi)(N^L)^{\alpha}\right]^{\frac{1-\alpha}{\alpha}} \left[\lambda\, K^{\gamma} + (1-\lambda)(N^H)^{\gamma}\right]^{\frac{\alpha-\gamma}{\gamma}} (N^H)^{\gamma-1} \tag{E.12}$$

$$F_N^L = (1-\phi)A\left[\phi\left[\lambda\, K^{\gamma} + (1-\lambda)(N^H)^{\gamma}\right]^{\frac{\alpha}{\gamma}} + (1-\phi)(N^L)^{\alpha}\right]^{\frac{1-\alpha}{\alpha}} (N^L)^{\alpha-1} \tag{E.13}$$

which is already a function of K and N^k. These two variables in turn also depend on the exogenous parameters already present in the above expression, like $unemp^k, partic^k, \varphi^k, \beta$. There is just no closed-form analytical solutions to show how exactly K depends on these parameters, since solving the capital marginal product function can only be done numerically: see (D.100).

E.2.3 Labor supply

Using the worker's labor supply FOCs w.r.t. n^k (D.129)-(D.130):

$$\lambda^{n,k} = \beta\Big[\lambda^{c,k}w^k + (1-\sigma^k)\lambda^{n,k} - \Phi^k(l^k)^{-\xi}\Big]$$

$$\Phi^k(l^k)^{-\xi} = \lambda^{c,k}w^k + \left(1-\sigma^k-\frac{1}{\beta}\right)\lambda^{n,k}$$

$$\frac{\Phi^k(l^k)^{-\xi}}{\lambda^{c,k}} = w^k + \left(1-\sigma^k-\frac{1}{\beta}\right)\frac{\lambda^{n,k}}{\lambda^{c,k}} \tag{int1}$$

Then using the worker's labor supply FOCs wrt u^k (D.127)-(D.128), and substituting (int1) in:

$$\lambda^{n,k} = \frac{\Phi^H\left(l_t^k\right)^{-\xi} - \varkappa^k\lambda^{c,k}}{\mu^k} =$$

$$= \frac{\lambda^{c,k}w^k + \left(1-\sigma^H-\frac{1}{\beta}\right)\lambda^{n,k} - \varkappa^k\lambda^{c,k}}{\mu^k}$$

$$\left[1-\frac{1-\sigma^k-\frac{1}{\beta}}{\mu^k}\right]\lambda^{n,k} = \frac{\left(w^k-\varkappa^k\right)\lambda^{c,k}}{\mu^k}$$

$$\frac{\lambda^{n,k}}{\lambda^{c,k}} = \frac{w^k-\varkappa^k}{\mu^k+\sigma^k+\frac{1-\beta}{\beta}} \tag{int2}$$

Then plugging back (int2) into (int1) yields:

$$\frac{\Phi^k(l^k)^{-\xi}}{\lambda^{c,k}} = w^k - \left(\sigma^k+\frac{1-\beta}{\beta}\right)\frac{w^k-\varkappa^k}{\mu^k+\sigma^k+\frac{1-\beta}{\beta}} =$$

$$= w^k - \left(\sigma^k+\frac{1-\beta}{\beta}\right)\frac{w^k}{\mu^k+\sigma^k+\frac{1-\beta}{\beta}} + \left(\sigma^k+\frac{1-\beta}{\beta}\right)\frac{\varkappa^k}{\mu^k+\sigma^k+\frac{1-\beta}{\beta}} =$$

$$= w^k\left[1-\frac{\sigma^k+\frac{1-\beta}{\beta}}{\mu^k+\sigma^k+\frac{1-\beta}{\beta}}\right] + \varkappa^k\frac{\sigma^k+\frac{1-\beta}{\beta}}{\mu^k+\sigma^k+\frac{1-\beta}{\beta}} =$$

$$= w^k\frac{\mu^k}{\mu^k+\sigma^k+\frac{1-\beta}{\beta}} + \varkappa^k\frac{\sigma^k+\frac{1-\beta}{\beta}}{\mu^k+\sigma^k+\frac{1-\beta}{\beta}} \tag{int3}$$

Next, we plug in (E.7) into (int3), while also using that from $\mu^k = \sigma^k\left[\frac{1}{unemp^k}-1\right]$ we get $\mu^k+\sigma^k = \frac{\sigma^k}{unemp^k}$:

$$\frac{\Phi^k(l^k)^{-\xi}}{\lambda^{c,k}} = w^k\frac{\sigma^k\left[\frac{1}{unemp^k}-1\right]}{\frac{\sigma^k}{unemp^k}+\frac{1-\beta}{\beta}} + \varkappa^k\frac{\sigma^k+\frac{1-\beta}{\beta}}{\frac{\sigma^k}{unemp^k}+\frac{1-\beta}{\beta}} =$$

$$= {\color{red}w^k}\frac{\sigma^k(1-unemp^k)}{\sigma^k+\frac{1-\beta}{\beta}unemp^k} + \varkappa^k\frac{\left(\sigma^k+\frac{1-\beta}{\beta}\right)unemp^k}{\sigma^k+\frac{1-\beta}{\beta}unemp^k} \tag{E.14}$$

as well as into (int2):

$$\frac{\lambda^{n,k}}{\lambda^{c,k}} = \frac{w^k-\varkappa^k}{\mu^k+\sigma^k+\frac{1-\beta}{\beta}} =$$

$$= \frac{{\color{red}w^k}-\varkappa^k}{\frac{\sigma^k}{unemp^k}+\frac{1-\beta}{\beta}} \tag{E.15}$$

Notice how (E.14) and (E.15) still include w^k, which could be substituted out using our earlier result (E.11).

E.2.4 Bargaining

Using the wage bargaining equation (D.131)-(D.132), we recover ϑ^k:

$$
\begin{aligned}
w^k &= \vartheta^k \left[x F_N^k + (1-\sigma^k)\frac{\kappa^k}{\nu^k} \right] + \frac{1-\vartheta^k}{\lambda^{c,k}} \left[\Phi^k \left(l^k\right)^{-\xi} - (1-\sigma^k)\lambda^{n,k} \right] = \\
&= \vartheta^k \left[x F_N^k + (1-\sigma^k)\frac{\kappa^k}{\nu^k} \right] + \frac{1}{\lambda^{c,k}} \left[\Phi^k \left(l^k\right)^{-\xi} - (1-\sigma^k)\lambda^{n,k} \right] - \frac{\vartheta^k}{\lambda^{c,k}} \left[\Phi^k \left(l^k\right)^{-\xi} - (1-\sigma^k)\lambda^{n,k} \right]
\end{aligned}
$$

$$
w^k - \frac{1}{\lambda^{c,k}} \left[\Phi^k \left(l^k\right)^{-\xi} - (1-\sigma^k)\lambda^{n,k} \right] = \vartheta^k \left\{ \left[x F_N^k + (1-\sigma^k)\frac{\kappa^k}{\nu^k} \right] - \frac{1}{\lambda^{c,k}} \left[\Phi^k \left(l^k\right)^{-\xi} - (1-\sigma^k)\lambda^{n,k} \right] \right\}
$$

$$
\begin{aligned}
\vartheta^k &= \frac{w^k - \frac{1}{\lambda^{c,k}} \left[\Phi^k \left(l^k\right)^{-\xi} - (1-\sigma^k)\lambda^{n,k} \right]}{\left[x F_N^k + (1-\sigma^k)\frac{\kappa^k}{\nu^k} \right] - \frac{1}{\lambda^{c,k}} \left[\Phi^k (l^k)^{-\xi} - (1-\sigma^k)\lambda^{n,k} \right]} = \\
&= \frac{w^k - \frac{\Phi^k (l^k)^{-\xi}}{\lambda^{c,k}} + (1-\sigma^k)\frac{\lambda^{n,k}}{\lambda^{c,k}}}{\left[x F_N^k + (1-\sigma^k)\frac{\kappa^k}{\nu^k} \right] - \frac{\Phi^k (l^k)^{-\xi}}{\lambda^{c,k}} + (1-\sigma^k)\frac{\lambda^{n,k}}{\lambda^{c,k}}} \qquad \text{(E.16)}
\end{aligned}
$$

where the red expressions can further be substituted out using (E.8), (E.11), (E.12)-(E.13), (E.14), (E.15).

E.2.5 Coefficients in the log-linear wage bargaining equation

By means of the above results for the steady-state values, we can rewrite the coefficients of (E.6) as functions of exogenous parameters/targets:

$$
\begin{aligned}
\widehat{w}_t^k = &\underbrace{\frac{\vartheta^k x F_N^k}{w^k}}_{\alpha_x^k} \widehat{x}_t + \underbrace{\frac{\vartheta^k x F_N^k}{w^k}}_{\alpha_{F_N}^k} \widehat{F}_{N,t}^k + \\
&+ \underbrace{\frac{1-\sigma^k}{w^k} \left[(1-\varsigma)\frac{\vartheta^k \kappa^k}{\nu^k} + \varsigma \frac{(1-\vartheta^k)\lambda^{n,k}}{\lambda^{c,k}} \right]}_{\alpha_\theta^k} \widehat{\theta}_t^k + \\
&+ \underbrace{\frac{\eta(1-\vartheta^k)}{w^k} \left[\frac{\Phi^k (l^k)^{-\xi}}{\lambda^{c,k}} - (1-\sigma^k) \left(\frac{\lambda^{n,k}}{\lambda^{ck}} + \frac{\varkappa^k}{\mu^k} \right) \right]}_{\alpha_c^k} \widehat{c}_t^k + \\
&+ \underbrace{\xi \, \frac{(1-\vartheta^k)\, \Phi^k (l^k)^{-\xi}}{\lambda^{c,k}\, w^k} \left[\frac{(1-\sigma^k)}{\mu^k} - 1 \right]}_{\alpha_l^k} \widehat{l}_t^k = \qquad \text{(E.6)} \\
= &\alpha_x^k \, \widehat{x}_t + \alpha_{F_N}^k \, \widehat{F}_{N,t}^k + \alpha_\theta^k \, \widehat{\theta}_t^k + \alpha_c^k \, \widehat{c}_t^k + \alpha_l^k \, \widehat{l}_t^k \qquad \text{(E.17)}
\end{aligned}
$$

where

- $x = 1$
- $\mu^k = \sigma^k \left[\frac{1}{unemp^k} - 1 \right]$ is given by (E.7)
- $\nu^k = (\psi^k)^{\frac{1}{\varsigma}} \left[\sigma^k \left(\frac{1}{unemp^k} - 1 \right) \right]^{\frac{\varsigma - 1}{\varsigma}}$ is given by (E.8)
- $w^k = x \, F_N^k - \kappa^k \left(\sigma^k + \frac{1-\beta}{\beta} \right) (\psi^k)^{-\frac{1}{\varsigma}} \left[\sigma^k \left(\frac{1}{unemp^k} - 1 \right) \right]^{\frac{1-\varsigma}{\varsigma}}$ is given by (E.11)
- $F_N^H = \phi(1-\lambda) \left[\phi \left[\lambda K^\gamma + (1-\lambda)(N^H)^\gamma \right]^{\frac{\alpha}{\gamma}} + (1-\phi)(N^L)^\alpha \right]^{\frac{1-\alpha}{\alpha}} \left[\lambda K^\gamma + (1-\lambda)(N^H)^\gamma \right]^{\frac{\alpha-\gamma}{\gamma}} (N^H)^{\gamma - 1}$ is given by (E.12) – for $k = L$ by (E.13)
- $\frac{\Phi^k (l^k)^{-\xi}}{\lambda^{c,k}} = w^k \, \frac{\sigma^k (1-unemp^k)}{\sigma^k + \frac{1-\beta}{\beta} unemp^k} + \varkappa^k \, \frac{\left(\sigma^k + \frac{1-\beta}{\beta} \right) unemp^k}{\sigma^k + \frac{1-\beta}{\beta} unemp^k}$ is given by (E.14)

- $\frac{\lambda^{n,k}}{\lambda^{c,k}} = \frac{w^k - \varkappa^k}{\frac{\sigma^k}{unemp^k} + \frac{1-\beta}{\beta}}$ is given by (E.15)
- $\vartheta^k = \frac{w^k - \frac{\Phi^k(l^k)^{-\xi}}{\lambda^{c,k}} + (1-\sigma^k)\frac{\lambda^{n,k}}{\lambda^{c,k}}}{\left[xF_N^k + (1-\sigma^k)\frac{\kappa^k}{\nu^k}\right] - \frac{\Phi^k(l^k)^{-\xi}}{\lambda^{c,k}} + (1-\sigma^k)\frac{\lambda^{n,k}}{\lambda^{c,k}}}$ is given by (E.16)

which verifies that the coefficients α^k can be expressed solely as functions of exogenous parameters and steady-state targets, i.e. all endogenous steady-state values (like Φ^k or w^k) can be substituted out. The analytical expressions would be quite complicated though.[4]

[4]However, an important caveat still applies: F_N^k could not be fully expressed as a function of exogenous steady-state targets in closed analytical form (due to our complex CSC production function), so F_N^k will remain part of the expressions, which itself numerically depends on exogenous parameters and steady-state targets.

E.3 Decomposing dynamics

Now we decompose the dynamics of real wage $\widehat{w}_t^k$ and the wage premium $\widehat{w}_t^H - \widehat{w}_t^L$ based on the log-linearized equations (E.17) and (E.18):

$$\widehat{w}_t^k = \alpha_x^k\, \widehat{x}_t \,+\, \alpha_{F_N}^k\, \widehat{F}_{N,t}^k \,+\, \alpha_\theta^k\, \widehat{\theta}_t^k \,+\, \alpha_c^k\, \widehat{c}_t^k \,+\, \alpha_l^k\, \widehat{l}_t^k \qquad \text{[E.17]}$$

$$\begin{aligned} \widehat{w}_t^H - \widehat{w}_t^L = &\left(\alpha_x^H - \alpha_x^L\right) \widehat{x}_t \,+\, \left[\alpha_{F_N}^H\, \widehat{F}_{N,t}^H - \alpha_{F_N}^L\, \widehat{F}_{N,t}^L\right] \,+\, \left[\alpha_\theta^H\, \widehat{\theta}_t^H - \alpha_\theta^L\, \widehat{\theta}_t^L\right] + \\ &+ \left[\alpha_c^H\, \widehat{c}_t^H - \alpha_c^L\, \widehat{c}_t^L\right] \,+\, \left[\alpha_l^H\, \widehat{l}_t^H - \alpha_l^L\, \widehat{l}_t^L\right] \end{aligned} \qquad \text{(E.18)}$$

where the blue terms have to do with adjustment from the firm's side (labor demand, firm's surplus), while the red terms have to do with adjustment from the household's side (labor supply, worker's surplus). It can be shown that labor market tightness contributes equally through the firm's (vacancy filling rate) and the household's (job finding rate) side, with $\alpha_{\theta,f}^k = \alpha_{\theta,h}^k$:

$$\alpha_\theta^k\, \widehat{\theta}_t^k = \left(\alpha_{\theta,f}^k + \alpha_{\theta,h}^k\right) \widehat{\theta}_t^k$$

Below we show how, in response to a monetary expansion, movements in each of these 5 variables contribute to the dynamics (IRF) of real wage and the wage premium. This is the product of their own IRFs (e.g. $\widehat{x}_t$) and their corresponding coefficients (e.g. α_x^k) in the above equations, so each colored bar in the charts represent a term like $\alpha_x^k\, \widehat{x}_t$.

E.3.1 Baseline: asym SAM + CSC

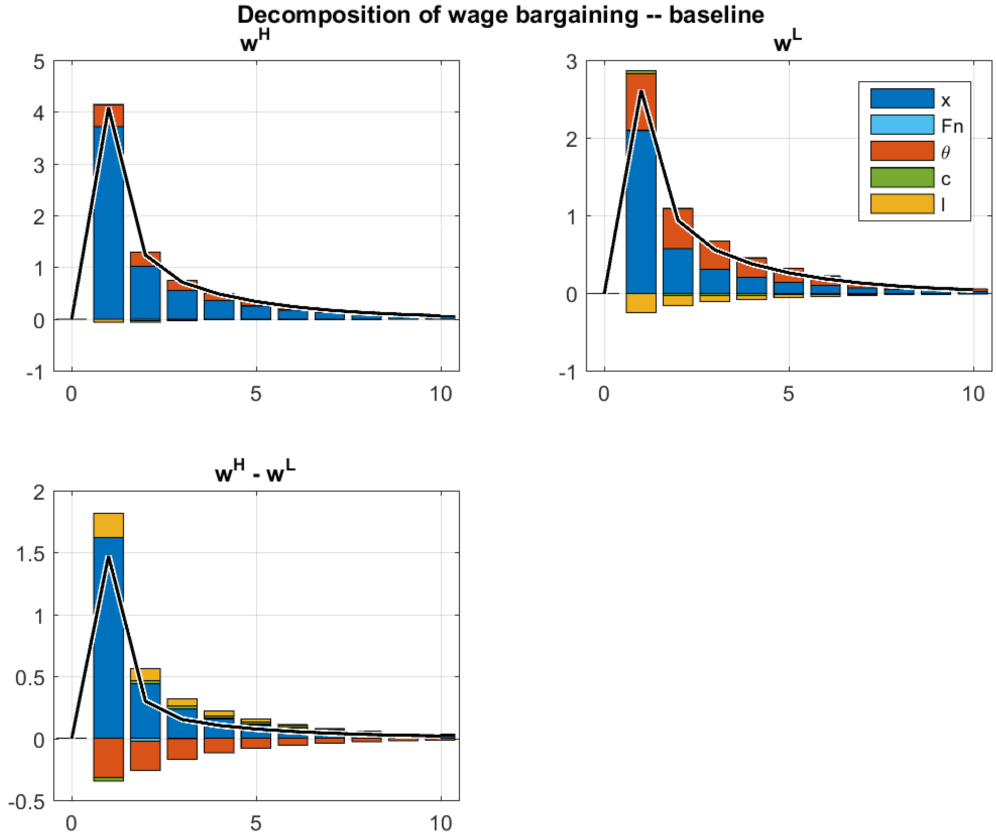


Figure 28: Decomposing real wage $\widehat{w}_t^k$ and wage premium $\widehat{w}_t^H - \widehat{w}_t^L$ dynamics based on equations (E.17) and (E.18).

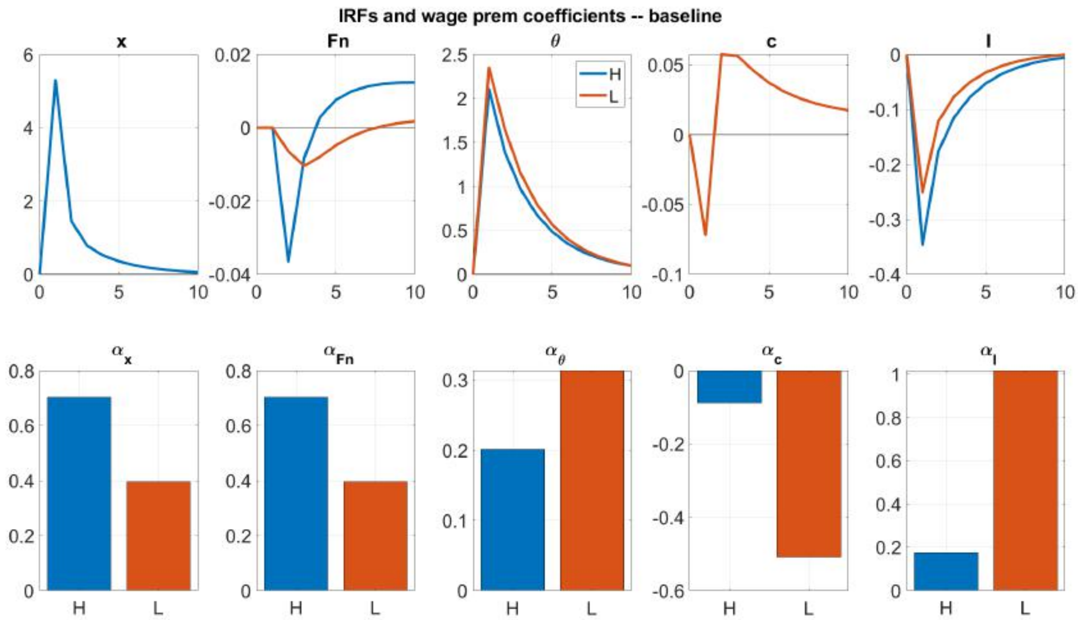


Figure 29: Impulse responses of the components of equation (E.17) and their coefficients α^k.

E.3.2 sym SAM + CD

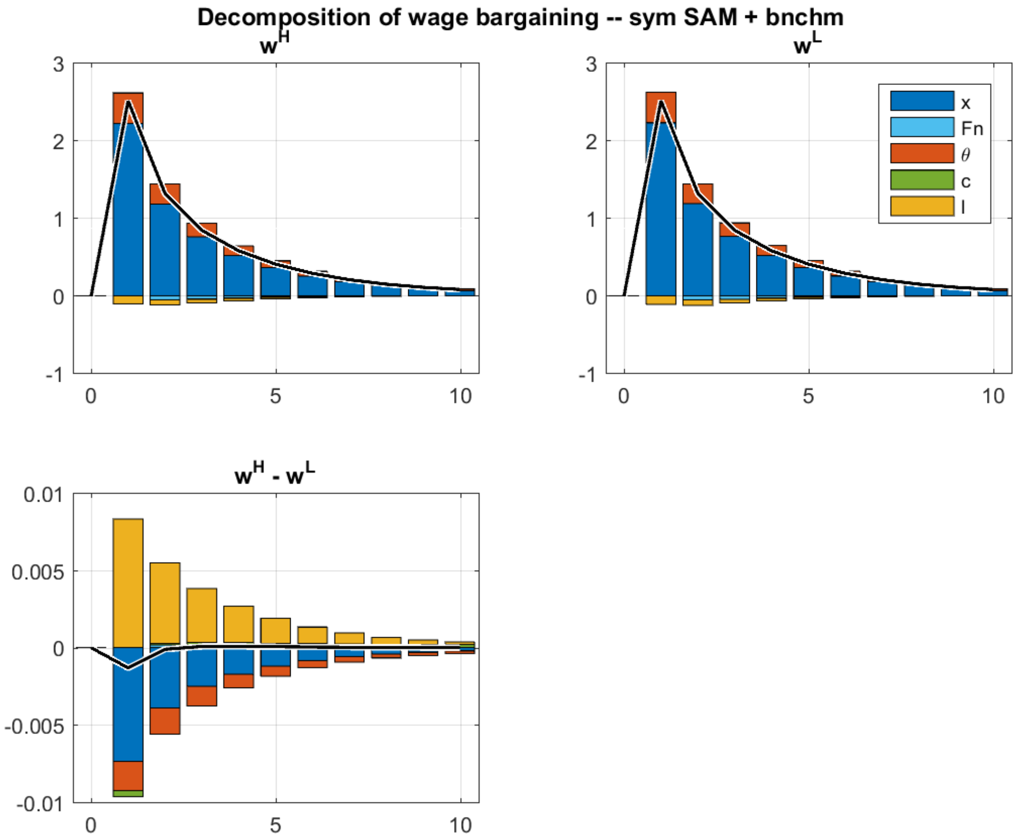


Figure 30: Decomposing real wage $\widehat{w}_t^k$ and wage premium $\widehat{w}_t^H - \widehat{w}_t^L$ dynamics based on equations (E.17) and (E.18).
Scenario: "Symmetric SAM + Cobb-Douglas production"

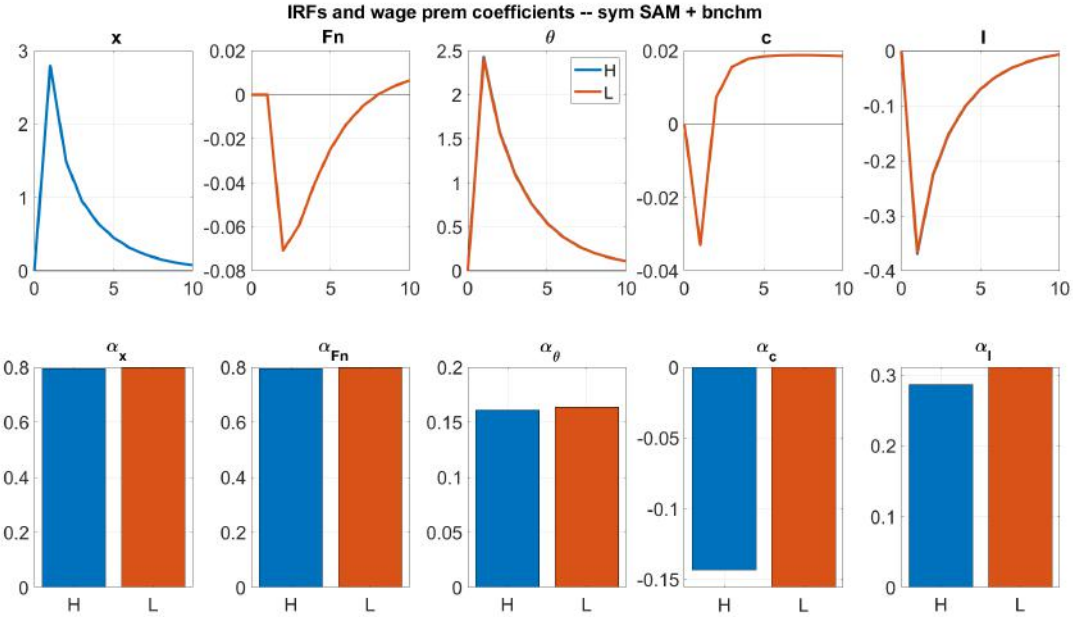


Figure 31: Impulse responses of the components of equation (E.17) and their coefficients α^k.
Scenario: "Symmetric SAM + Cobb-Douglas production"

E.3.3 sym SAM + CSC

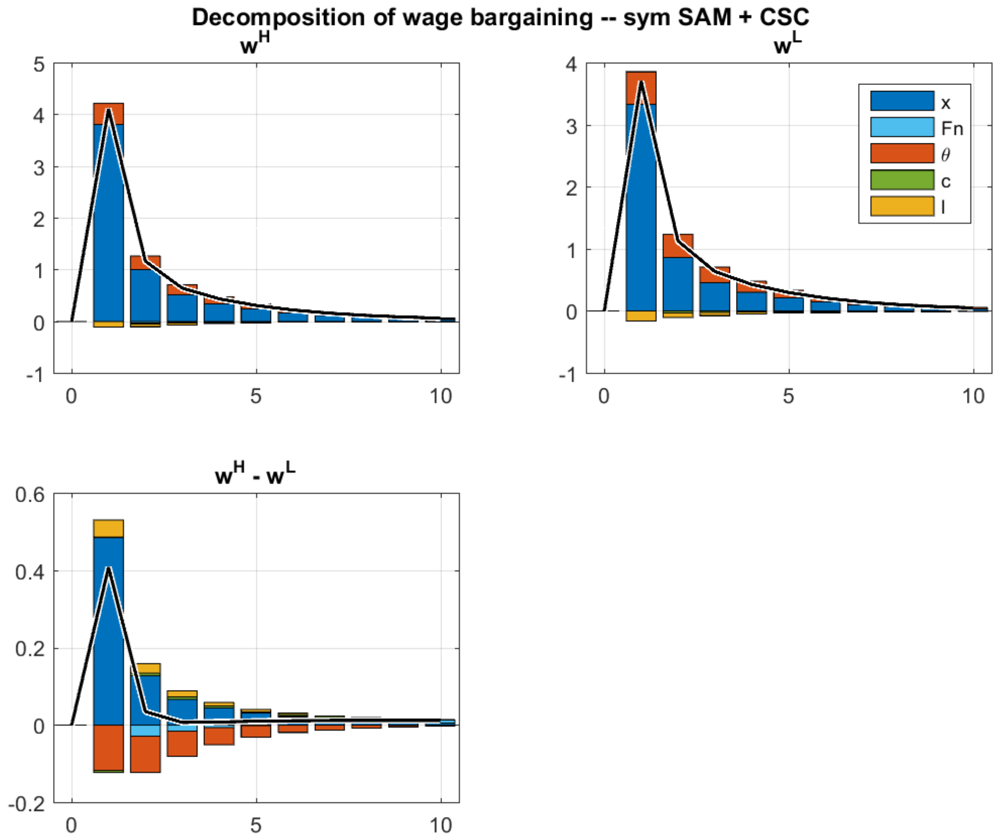


Figure 32: Decomposing real wage $\widehat{w}_t^k$ and wage premium $\widehat{w}_t^H - \widehat{w}_t^L$ dynamics based on equations (E.17) and (E.18). **Scenario: "Symmetric SAM + CSC production"**

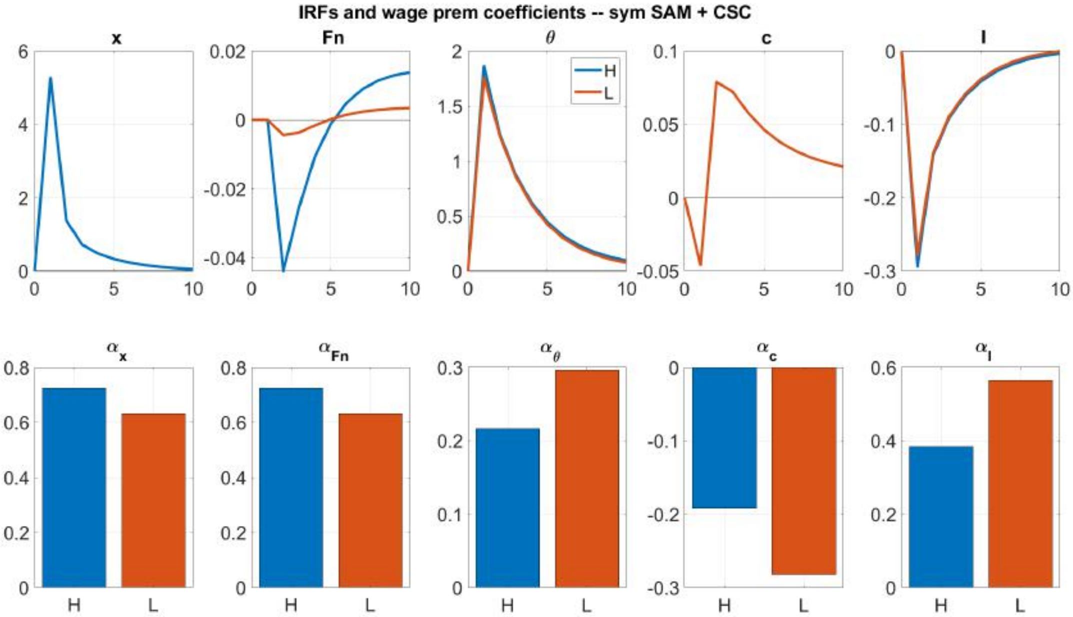


Figure 33: Impulse responses of the components of equation (E.17) and their coefficients α^k.
Scenario: "Symmetric SAM + CSC production"

E.3.4 asym SAM + CD

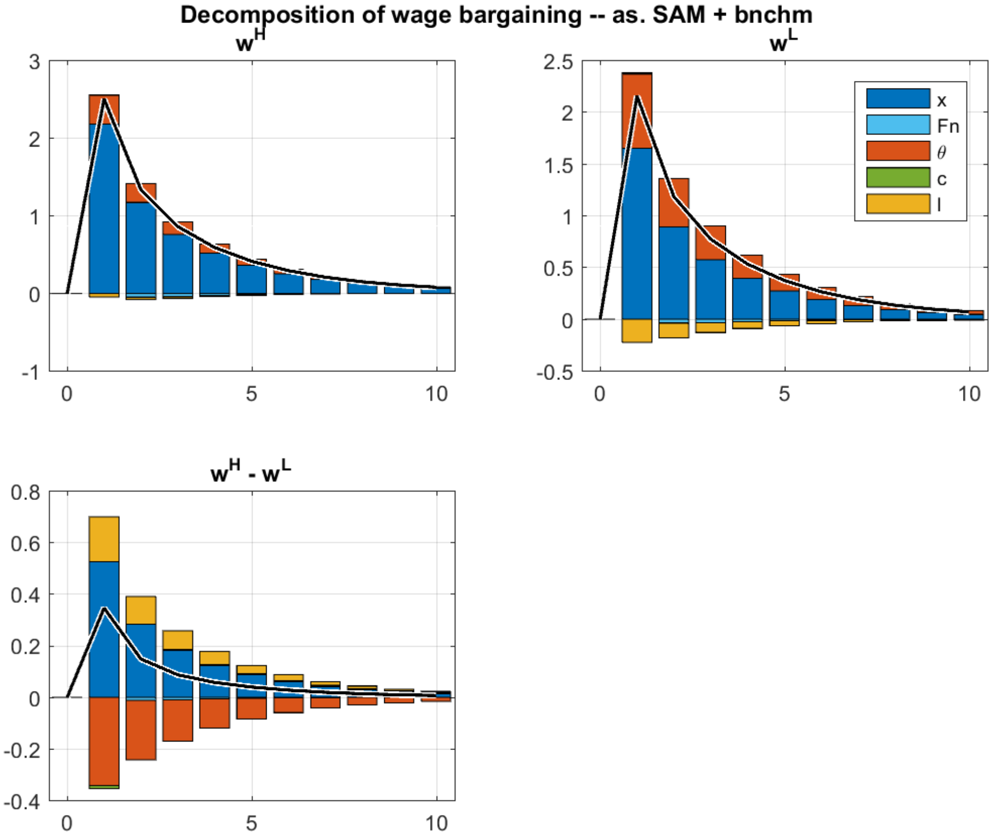


Figure 34: Decomposing real wage $\widehat{w}_t^k$ and wage premium $\widehat{w}_t^H - \widehat{w}_t^L$ dynamics based on equations (E.17) and (E.18). **Scenario: "Asymmetric SAM + Cobb-Douglas production"**

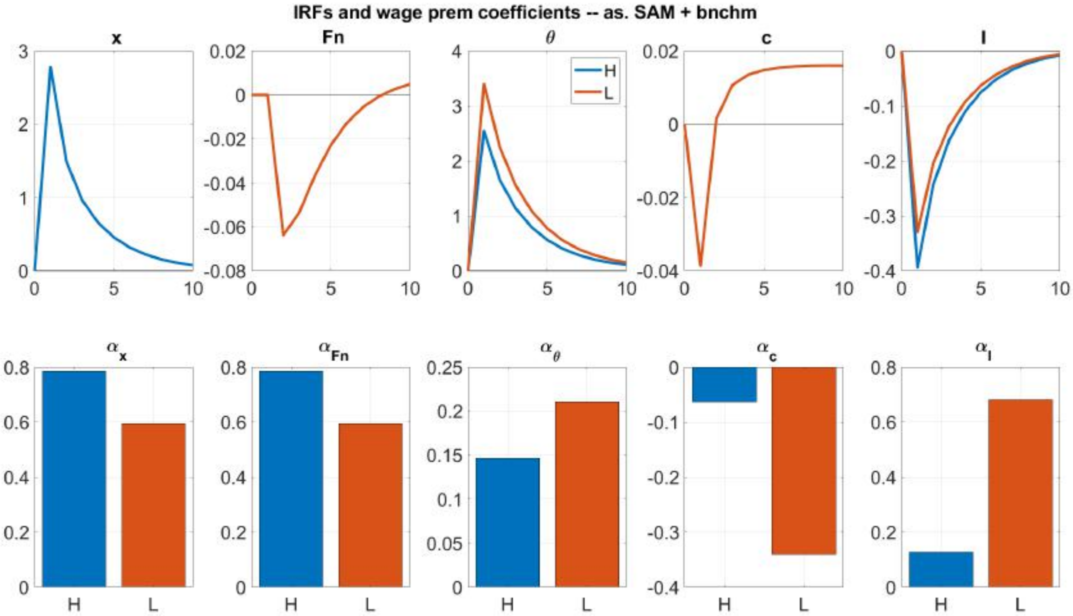


Figure 35: Impulse responses of the components of equation (E.17) and their coefficients α^k.
Scenario: "Asymmetric SAM + Cobb-Douglas production"

E.3.5 Variable capital utilization (asym SAM + CSC)

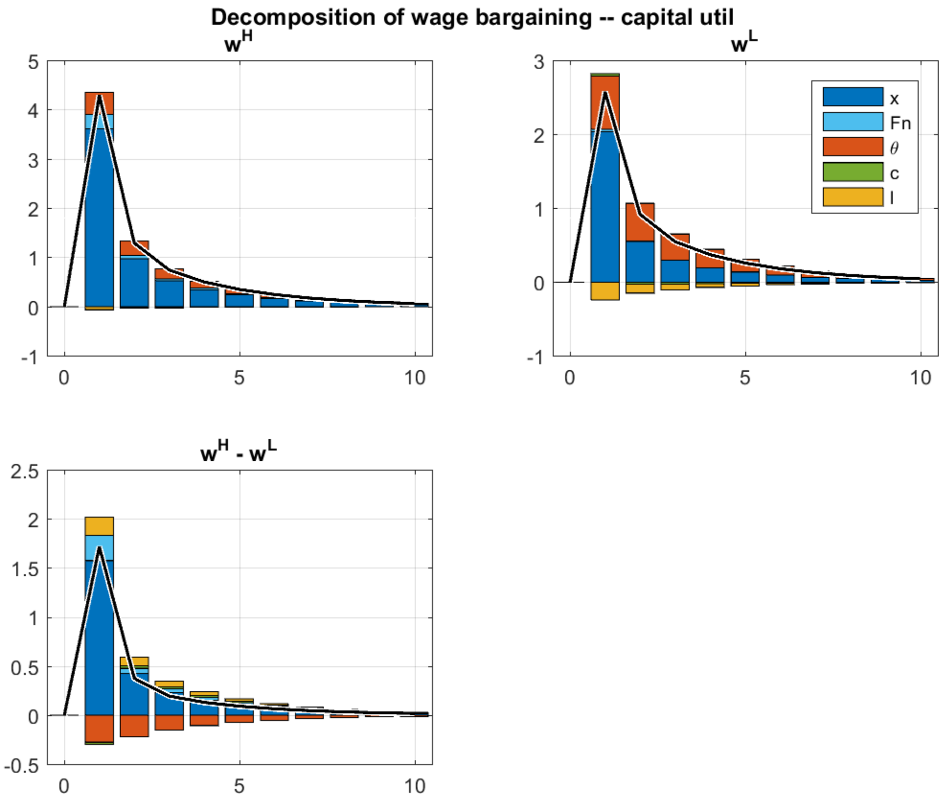


Figure 36: Decomposing real wage $\widehat{w}_t^k$ and wage premium $\widehat{w}_t^H - \widehat{w}_t^L$ dynamics based on equations (E.17) and (E.18).
Scenario: "variable capital utilization"

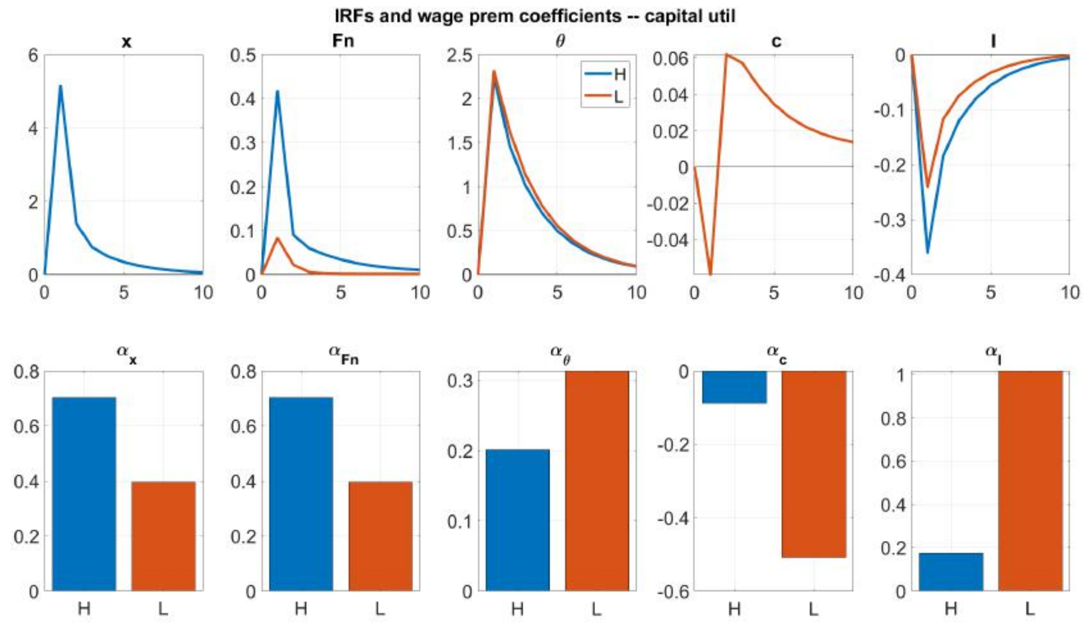


Figure 37: Impulse responses of the components of equation (E.17) and their coefficients α^k.
Scenario: "variable capital utilization"

E.3.6 Comparisons across scenarios (non-variable capital utilization)

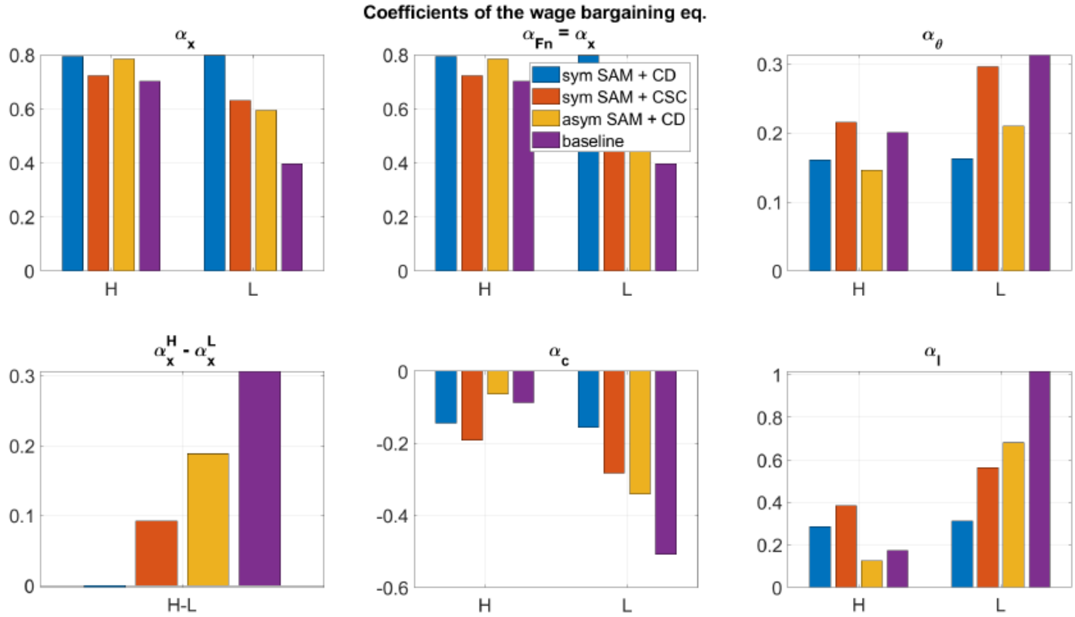


Figure 38: Comparing the coefficients α^k of equation (E.17) across different scenarios.

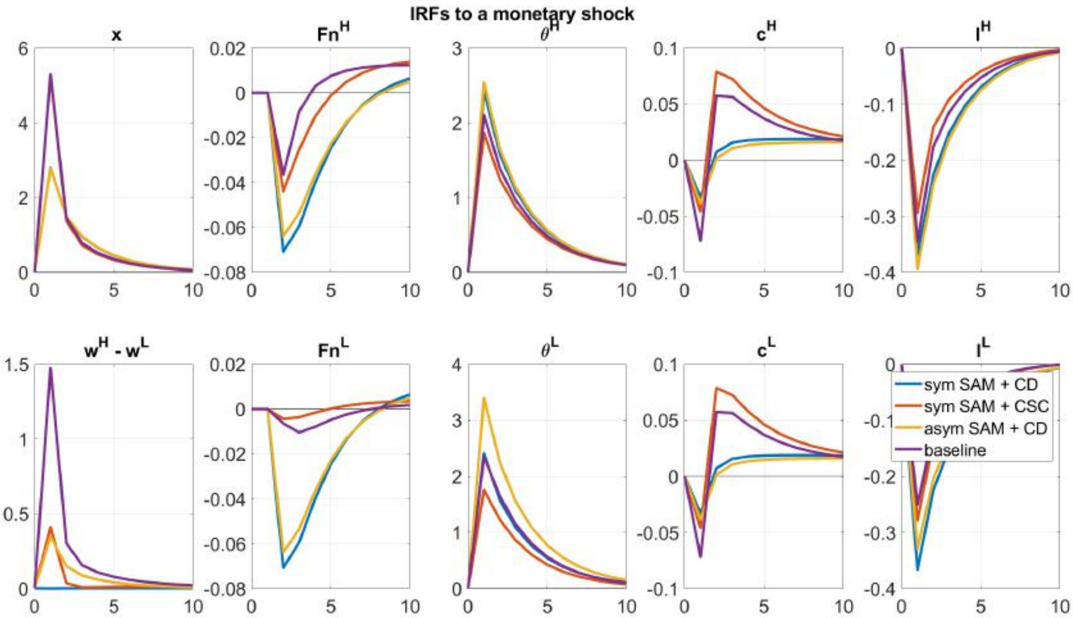


Figure 39: Comparing the IRFs of variables in equation (E.17) across different scenarios.

E.3.7 Comparisons across scenarios (variable capital utilization)

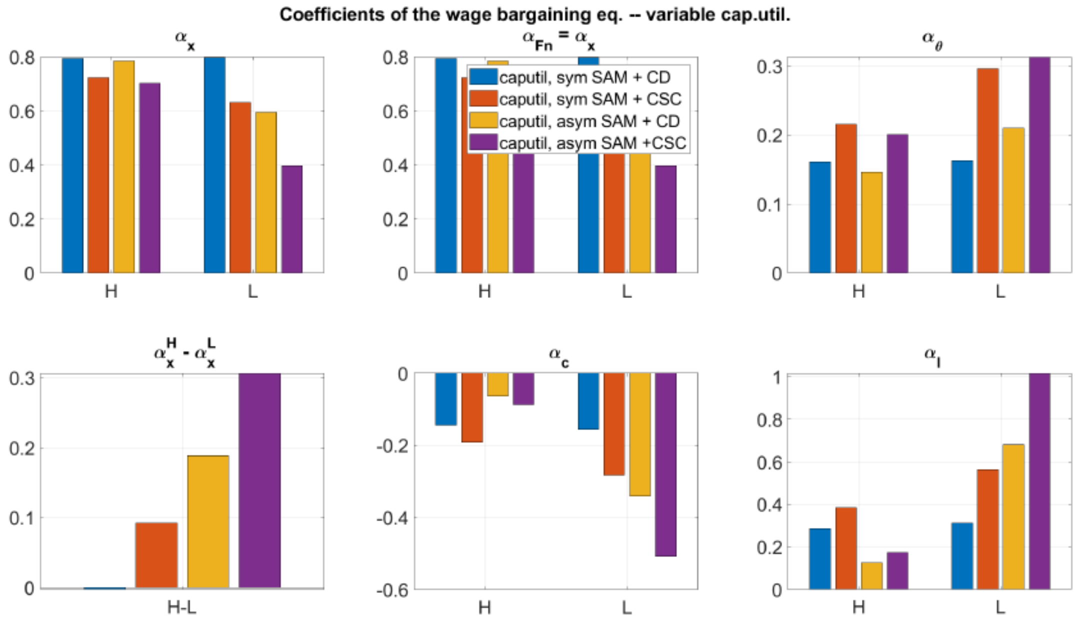


Figure 40: Comparing the coefficients α^k of equation (E.17) across different scenarios.

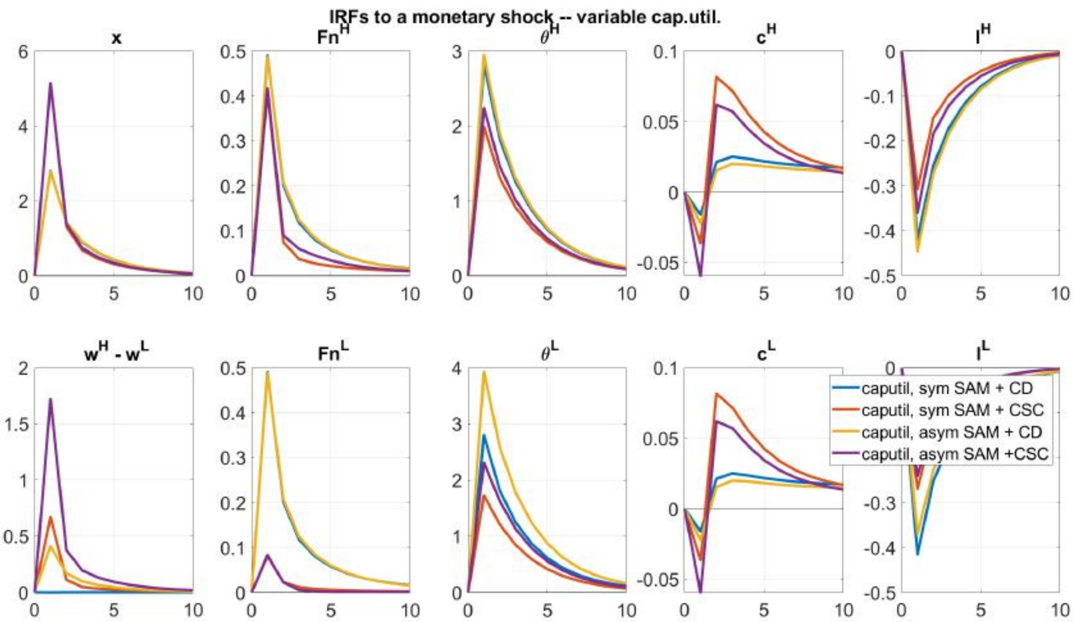


Figure 41: Comparing the IRFs of variables in equation (E.17) across different scenarios.

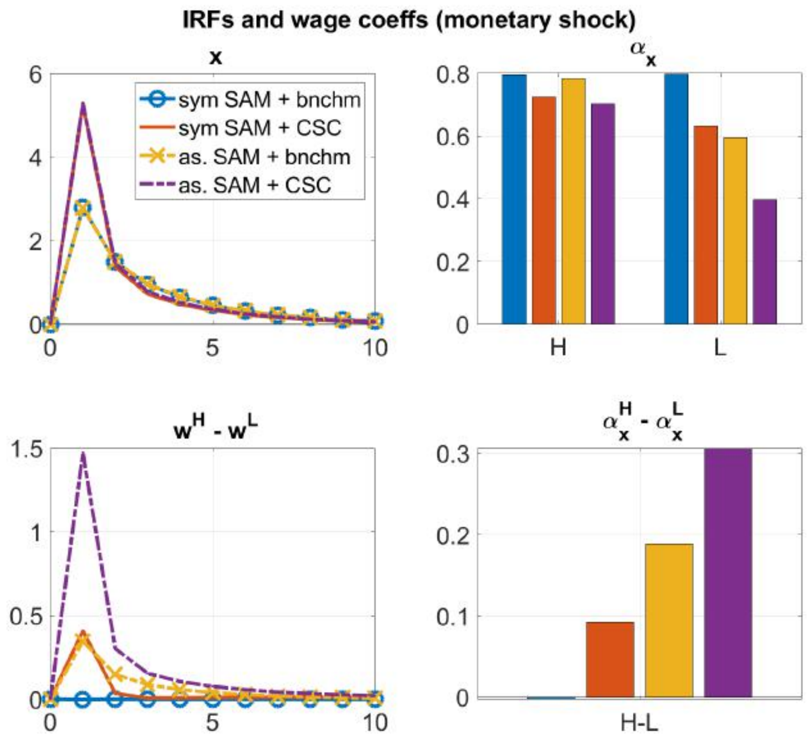


Figure 42: Comparing the IRFs of variables in equation (E.17) across different scenarios – with **non-variable capital utilization.**

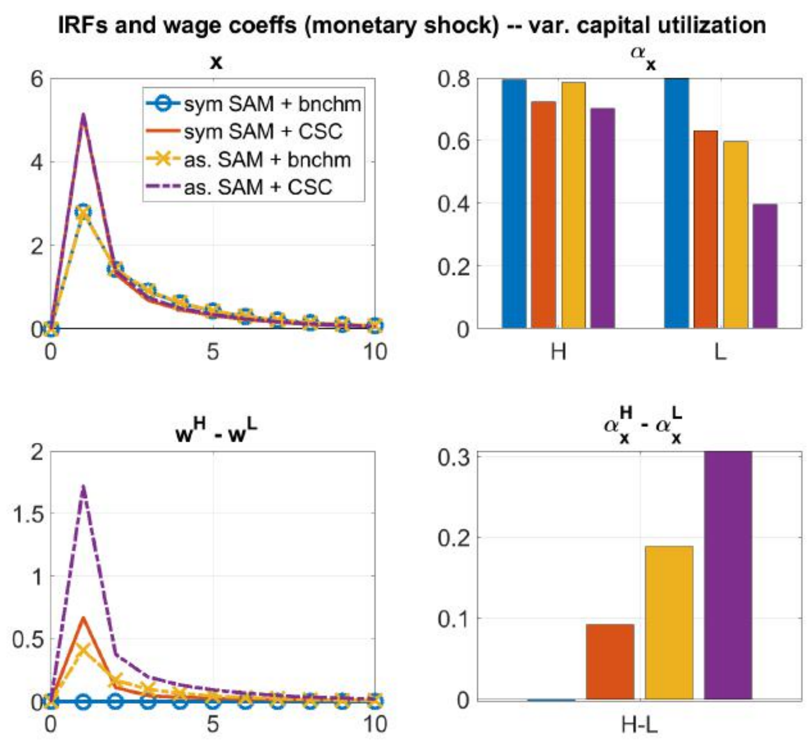


Figure 43: Comparing the IRFs of variables in equation (E.17) across different scenarios – with **variable capital utilization**.

F Further details on some derivations

F.1 Retail firms

The FOC of the problem of the retail firms is derived as follows. Due to symmetry and homogeneity across retailers, all of them will choose the same price $p_t^*(i) = p_t^*$. The solution to the problem yields:

$$E_t \sum_{s=0}^{\infty} \chi^s \, \Lambda_{t+s} \left[(1-\epsilon) \underbrace{p_t(i)^{-\epsilon} P_{t+s}^{\epsilon} Y_{t+s}}_{y_{t+s}(i)} \frac{1}{P_{t+s}} - (1-\tau) x_{t+s} (-\epsilon) \underbrace{p_t(i)^{-\epsilon-1} P_{t+s}^{\epsilon} Y_{t+s}}_{\frac{y_{t+s}(i)}{p_t(i)}} \right] = 0$$

$$E_t \sum_{s=0}^{\infty} \chi^s \, \Lambda_{t+s} \, y_{t+s}(i) \left[p_t(i) - \frac{(1-\tau)\epsilon}{\epsilon - 1} P_{t+s} x_{t+s} \right] = 0$$

$$p_t^* = \underbrace{\frac{(1-\tau)\epsilon}{\epsilon-1}}_{(1-\tau)\mathcal{M}} E_t \frac{\sum_{s=0}^{\infty} \chi^s \, \Lambda_{t+s} \, y_{t+s}(i) \, \overbrace{P_{t+s} x_{t+s}}^{MC_t}}{\sum_{s=0}^{\infty} \chi^s \, \Lambda_{t+s} \, y_{t+s}(i)}$$

which yields the expression displayed in the main text. Retailers charge a markup over some discounted value of future marginal costs.

The recursive formulation of this pricing decision is derived as follows. Divide the FOC of the retailers by $\Pi_{t,t+s} = \frac{P_{t+s}}{P_t}$ so that we get a nominal discount factor and substitute in for Λ_{t+s} (defining $\tilde{c}_t \equiv c_t^E - h\, c_{t-1}^E$, and $O_s = \prod_{k=1}^{s} \Omega_{t+k-1}$) and the demand function $Y_{t+s}(i)$:

$$E_t \sum_{s=0}^{\infty} \chi^s \, \Lambda_{t+s} \, Y_{t+s}(i) \left[p_t(i) - \frac{(1-\tau)\epsilon}{\epsilon - 1} P_{t+s} x_{t+s} \right] = 0$$

$$E_t \sum_{s=0}^{\infty} \chi^s \underbrace{\beta^s O_s \left(\frac{\tilde{c}_{t+s}}{\tilde{c}_t} \right)^{-\eta}}_{\Lambda_{t+s}} \frac{1}{\Pi_{t,t+s}} \underbrace{\left(\frac{p_t^*}{P_{t+s}} \right)^{-\epsilon} Y_{t+s}}_{Y_{t+s}(i)} \left[p_t^* - \frac{(1-\tau)\epsilon}{\epsilon - 1} P_{t+s} x_{t+s} \right] = 0$$

Divide by the terms not dependent on s, which are $(\tilde{c}_t)^{\eta}$ and $(p_t^*)^{-\epsilon}$ and rearrange to get

$$E_t \sum_{s=0}^{\infty} \chi^s \beta^s O_s \, (\tilde{c}_{t+s})^{-\eta} \frac{1}{\Pi_{t,t+s}} \left(\frac{1}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \, p_t^* =$$

$$= E_t \sum_{s=0}^{\infty} \chi^s \beta^s O_s \, (\tilde{c}_{t+s})^{-\eta} \frac{1}{\Pi_{t,t+s}} \left(\frac{1}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \left[\frac{(1-\tau)\epsilon}{\epsilon - 1} P_{t+s} x_{t+s} \right]$$

Use $\frac{P_{t+s}}{\Pi_{t,t+s}} = P_t$ on the RHS, then multiply by $P_t^{-\epsilon}$:

$$E_t \sum_{s=0}^{\infty} \chi^s \beta^s O_s \, (\tilde{c}_{t+s})^{-\eta} \frac{1}{\Pi_{t,t+s}} \left(\frac{P_t}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \, p_t^* =$$

$$= \; E_t \sum_{s=0}^{\infty} \chi^s \beta^s O_s \, (\tilde{c}_{t+s})^{-\eta} P_t \left(\frac{P_t}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \left[\frac{(1-\tau)\epsilon}{\epsilon - 1} x_{t+s} \right]$$

$$E_t \sum_{s=0}^{\infty} \chi^s \beta^s O_s \, (\tilde{c}_{t+s})^{-\eta} \; Y_{t+s} \, \Pi_{t,t+s}^{\epsilon-1} \, p_t^* = E_t \sum_{s=0}^{\infty} \chi^s \beta^s O_s \, (\tilde{c}_{t+s})^{-\eta} \; Y_{t+s} \, \Pi_{t,t+s}^{\epsilon} \, P_t \left[\frac{(1-\tau)\epsilon}{\epsilon - 1} x_{t+s} \right]$$

$$\frac{p_t^*}{P_t} = \frac{(1-\tau)\epsilon}{\epsilon - 1} \; \frac{E_t \sum_{s=0}^{\infty} \chi^s \beta^s O_s \, (\tilde{c}_{t+s})^{-\eta} \; Y_{t+s} \, \Pi_{t,t+s}^{\epsilon} \, x_{t+s}}{E_t \sum_{s=0}^{\infty} \chi^s \beta^s O_s \, (\tilde{c}_{t+s})^{-\eta} \; Y_{t+s} \, \Pi_{t,t+s}^{\epsilon-1}} \equiv \frac{\Theta_t}{\Delta_t}$$

We also know that:

$$P_t^{1-\epsilon} = (1-\chi)(p_t^*)^{1-\epsilon} + \chi P_{t-1}^{1-\epsilon}$$

$$\frac{p_t^*}{P_t} = \frac{1}{\Pi_t} \left[\frac{1}{1-\chi} \left(\Pi_t^{1-\epsilon} - \chi \right) \right]^{\frac{1}{1-\epsilon}} =$$

$$= \frac{\Theta_t}{\Delta_t} = \left[\frac{1 - \chi \Pi_t^{\epsilon - 1}}{1-\chi} \right]^{\frac{1}{1-\epsilon}} \tag{F.1}$$

And now we express Δ_t and Θ_t recursively, also pluggin in for $\tilde{c}_t$:

$$\Theta_t = Y_t\,(c_t^E - hc_{t-1}^E)^{-\eta}\, x_t\, \frac{(1-\tau)\epsilon}{\epsilon - 1} \;+\; \chi\beta_t E_t\left[\Pi_{t+1}^{\epsilon}\Theta_{t+1}\right] \tag{F.2}$$

$$\Delta_t = Y_t\,(c_t^E - hc_{t-1}^E)^{-\eta} \;+\; \chi\beta_t E_t\left[\Pi_{t+1}^{\epsilon-1}\Delta_{t+1}\right] \tag{F.3}$$

The last three equations pin down gross inflation Π_t, given other real variables (c_t^E, Y_t, x_t) – this is a non-linear substitute for the log-linearized New Keynesian Phillips Curve. Real marginal cost x_t can be subject to ad-hoc exogenous cost-push shocks Ξ_t, in which case the above derivations must include the term $x_t\Xi_t$ instead of a sole x_t.

F.2 Market clearing

- **bond market** clears in nominal terms $\varphi^E B_t^E + \varphi^H B_t^H + \varphi^L B_t^L = 0$, from which it follows (after dividing by p_t) that it has to clear in real terms as well:

$$\varphi^E b_t^E + \varphi^H b_t^H + \varphi^L b_t^L = 0 \tag{F.4}$$

- the **goods market** automatically clears by Walras' Law: combining the household and government budget constraints and the above market clearing conditions will yield the resource constraint so it does not have to be imposed as a separate equilibrium condition.
- for intermediate goods firms, real costs plus profit should make up real intermediate output $x_tY_t = r_tK_t + w_tN_t + \kappa v_t + d_t^{int}$
- retail firms make a profit depending on the (time-varying) average markup, i.e. whenever the after-subsidy average real marginal cost is not equal to 1: $d_t^r = \big(1-(1-\tau)x_t\big)Y_t$. In steady state they should be getting the same profits with or without subsidy: it is either paid by the government through non-distortionary taxation or gained from distorting retail prices above the marginal cost:

$$\overline{d}^r = \left(1 - (1-\tau)\underbrace{\frac{\epsilon - 1}{\epsilon(1-\tau)}}_{\overline{x}}\right)\overline{Y} = \frac{1}{\epsilon}\overline{Y}$$

$$
\begin{aligned}
E:&\quad \varphi^E\left[c_t^E + i_t + t_t^E + b_t^E = r_t(z_tk_t) + \frac{R_{t-1}}{\Pi_t}b_{t-1}^E + d_t\right]\\
H:&\quad \varphi^H\left[c_t^H + t_t^H + b_t^H = w_t^H n_t^H + \frac{R_{t-1}}{\Pi_t}b_{t-1}^H + \varkappa^H u_t^H\right]\\
L:&\quad \varphi^L\left[c_t^L + t_t^L + b_t^L = w_t^L n_t^L + \frac{R_{t-1}}{\Pi_t}b_{t-1}^L + \varkappa^L u_t^L\right]\\
gov:&\quad G_t + \varkappa^H\varphi^H u_t^H + \varkappa^L\varphi^L u_t^L + \tau x_tY_t = \sum_{k\in E,H,L}\varphi^k t_t^k
\end{aligned}
$$

$$\underbrace{\sum_{k\in E,H,L}\varphi^k c_t^k}_{\equiv C_t} + \underbrace{\varphi^E i_t}_{\equiv I_t} + G_t + \tau x_tY_t + \underbrace{\sum_{k\in E,H,L}\varphi^k b_t^k}_{=0 \text{ by (F.4)}} = r_tK_t + w_t^H N_t^H + w_t^L N_t^L + \underbrace{\varphi^E d_t}_{\equiv d_t^{int} + d_t^r} + \frac{R_{t-1}}{\Pi_t}\underbrace{\sum_{k\in E,H,L}\varphi^k b_{t-1}^k}_{=0 \text{ by (F.4)}}$$

$$C_t + I_t + G_t + \tau x_tY_t = \underbrace{r_tK_t + w_tN_t + d_t^{int}}_{x_tY_t - \kappa v_t} + \underbrace{d_t^r}_{\left(\frac{p_t}{p_t} - (1-\tau)x_t\right)Y_t}$$

$$C_t + I_t + G_t + \kappa^H v_t^H + \kappa^L v_t^L = Y_t \tag{F.5}$$

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